

MSM3A05a/MSM4A05a Problem Sheet 2.

QUESTION 1. Find a three term expansion for the function $\sin 2x$ using the asymptotic sequence

$$\left\{ \frac{x}{(1-x^2)^{\frac{1}{2}}}, \frac{x^2}{(1-x^2)^{\frac{1}{2}}}, \frac{x^3}{(1-x^2)^{\frac{1}{2}}}, \dots \right\}$$

as $x \rightarrow 0$.

QUESTION 2.

Obtain the first three terms of the asymptotic expansion of the following integral for large x

$$\int_0^4 t^2 \sqrt{1+t} e^{-xt} dt.$$

QUESTION 3. Obtain the first three terms of the asymptotic expansion of the following integral for large x

$$\int_0^3 \frac{1}{\sqrt{t}} \ln(1+t^2) e^{-xt} dt.$$

QUESTION 4. Find the leading order asymptotic expression for the modified Bessel function

$$K_\nu(x) = \int_0^\infty e^{-x \cosh t} \cosh(\nu t) dt,$$

as $x \rightarrow \infty$.

QUESTION 5. Obtain the leading order asymptotic expansion of the following integral for large x

$$\int_{\pi/4}^{\pi/2} \cos(t) e^{-x \cosh t} dt.$$

JU 08/10/12.