Assessed Example Sheet 1. MSM3A05/MSM4A05
Due to be handed in at 10am Tuesday 1st November.

QUESTION 1. [3 MARKS] Arrange the following in descending order for small ϵ

$$\epsilon^{\nu},\ \ \epsilon^{-\mu},\ \ \ln\left(\frac{1}{\epsilon}\right),\ \ \epsilon^{-\nu},\ \ \epsilon,\ \ e^{-\frac{1}{\epsilon}},\ \ \epsilon^{\mu},$$

where $\nu = 10^{-100}$ and $\mu = 10^{100}$.

QUESTION 2. [6 MARKS] Find an asymptotic expansion of the function $\ln(1+x)$ using the sequence of functions $\{1, \sin x, \sin^2 x, \sin^3 x \cdots\}$ as $x \to 0$. That is find a_0, a_1, a_2 and a_3 where

$$\ln(1+x) = a_0 + a_1 \sin x + a_2 \sin^2 x + a_3 \sin^3 x + \cdots$$

QUESTION 3*. [12 MARKS] Use Laplace's method to show that the modified Bessel function $K_\nu(z)$, which has the integral representation

$$K_\nu(z) = \frac{1}{2}\int_{-\infty}^{\infty} e^{\nu t - z\cosh t} dt \qquad (1)$$

can be approximated as $\nu \to \infty$, with $z = O(1)$ and positive, using

$$K_\nu(z) \sim \sqrt{\frac{\pi}{2\nu}} e^{-\nu}\left(\frac{2\nu}{z}\right)^{\nu}. \qquad (2)$$

[Hint: First find the local maximum of the exponent in (1) and call this $t = t_{\max}$. You will then need to use the identity $\sinh^{-1} y = \ln\left(y + \sqrt{1+y^2}\right)$ to find a suitable representation of $t_{\max}$. Then use Laplace's method as in the notes to find (2).]

QUESTION 4. MARKS Use Watson's lemma to determine

$$\int_0^{\infty} e^{-xs}\left(1 + \frac{is}{5}\right)^{-\frac{1}{2}} ds \qquad x \to \infty.$$

$*$ denotes a difficult question.

JU 15/10/12