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Title: A quantitative version of the commutator theorem for zero trace matrices

Abstract: As is well known, a complex $m \times m$ matrix A is a commutator (i.e., there are matrices B and C of the same dimensions as A such that $A = [B, C] = BC - CB$) if and only if A has zero trace. In such a situation clearly $\|A\| \leq 2\|B\|\|C\|$ where $\|D\|$ denotes the norm of D as an operator from ℓ_2^m to itself.

Does the converse hold? That is, if A has zero trace are there $m \times m$ matrices B and C such that $A = [B, C]$ and $\|B\|\|C\| \leq K\|A\|$ for some absolute constant K? If not, what is the behavior of the best K as a function of m?

I'll present some new insight, due to Johnson, Ozawa and myself concerning this question.