

- (1) The Hahn Banach theorem, its Corollary.
- (2) Baire Category Theorem.
- (3) Principle of nested balls.
- (4) Equivalent statement of BCT about closed sets with union equal the whole space.
- (5) Sets of 1st and 2nd category: definition, basic properties.
- (6) Complete metric space, Banach space are of 2nd category.
- (7) There exist a continuous function $f : [0, 1] \rightarrow \mathbb{R}$ which is nowhere differentiable. The set of such functions is dense in $C[0, 1]$ in supremum norm.
- (8) Principle of uniform boundedness for functions.
- (9) Family of linear operators: pointwise bounded, uniformly bounded.
- (10) Banach-Steinhaus Theorem, Principle of uniform boundedness for operators.
- (11) Principle of condensation of singularities.
- (12) Limit of a uniformly bounded sequence of linear operators that converge on a dense subset.
- (13) There exists a 2π -periodic continuous function from \mathbb{R} to \mathbb{R} such that its Fourier series diverges at zero.
- (14) The norm of the linear bounded functional defined as integral over a finite interval with continuous kernel.
- (15) Approximation of integrals.
- (16) Uniformly distributed sequences.
- (17) Weyl Theorem, examples of uniformly distributed sequences.
- (18) Open mapping, basic properties. Image of any complete space is complete.
- (19) Open mapping Theorem.
- (20) Inverse mapping Theorem.
- (21) Direct sum of Banach spaces, graph of a mapping.
- (22) Equivalent conditions for closed graph.
- (23) Closed graph Theorem.
- (24) Topological vector spaces, seminorms, locally convex spaces.
- (25) Examples of LCS. Weak and weak-star topologies.
- (26) Metrizable and non-metrizable LCS.
- (27) Example of a non-metrizable LCS.
- (28) Weak and weak-star convergence. Connections with convergence in norm.
- (29) Hyperplanes, theorems about separation of two sets in a LCS.

- (30) A convex subset of a normed space is closed if and only if it is weakly closed.
- (31) Infinite-dimensional sphere is not weakly closed; its weak closure contains zero.
- (32) Alaoglu's Theorem.
- (33) Convergence in coordinates of a subsequence of a sequence of points in $\mathbb{R}^{\mathbb{N}}$.
- (34) The unit ball of normed space is weak-star dense in the unit ball of its second dual.

Not examinable X is reflexive if and only if X^* is reflexive.

- (35) Compact operators: definition, basic properties.
- (36) Equicontinuous family of functions.
- (37) Arzela-Ascoli's Theorem.
- (38) Schauder Theorem.
- (39) Compact operators: linear combination, composition with bounded operator, limit give a compact operator.
- (40) Finite rank operators, approximation of a compact operator by finite rank operators.
- (41) Any Hilbert space has the approximation property.
- (42) Hilbert Schmidt operators.
- (43) Properties of $T = I - U$, where U is a compact operator; equivalence of T being invertible and T being injective and T being surjective.
- (44) Fredholm Alternative.
- (45) Spectral theorem of compact operators, zero is always in the spectrum of a compact operator on an infinite-dimensional space.