PROBLEM SHEET 0.

Exercise 1. Let X be a vector space and $q : X \to \mathbb{R}$ be a sublinear functional. Show that $q(0) = 0$ and $q(-x) \geq -q(x)$ for all $x \in X$.

Exercise 2. Show that $q : \ell^\infty \to \mathbb{R}$ defined as $q(x) = \limsup_{n\to\infty} x_n$ is a sublinear functional. (Here ℓ^∞ is the space of bounded sequences of real numbers.)

Exercise 3. Use the validity of Corollary 1.2 in the case when X is a real normed space in order to prove its statement in case X is a complex normed space:

Let X be a normed space over $\mathbb{C}$, Y be a linear subspace of X and $\psi \in Y^*$. Then there exists $\varphi \in X^*$ such that φ and ψ coincide on Y and $\|\varphi\|_{X^*} = \|\psi\|_{Y^*}$.

Exercise 4. Let X be a real normed space and let Y be a linear subspace of X. Suppose that $U : Y \to \mathbb{R}^n$ is a bounded linear operator. Prove that there exists a bounded linear extension V of U on the whole space X.