# Lecture 2.

**Proposition 1.**
Assume $\alpha \in (0,1)$ and $a, b > 0$. Then

$$a^\alpha b^{1-\alpha} \le \alpha a + (1-\alpha) b \tag{1}$$

and the equality is achieved if and only if $a = b$.
**Proposition 2.** **Young's inequality**
If $p > 1$, $p'$ is such that

$$\frac{1}{p} + \frac{1}{p'} = 1$$

(that is, $p' = p/(p-1)$), and $a$ and $b$ are positive real numbers, then

$$ab \le \frac{a^p}{p} + \frac{b^{p'}}{p'}, \tag{2}$$

where equality achieved if and only if $a^p = b^{p'}$.

## Hölder's Inequality

If $p > 1$ and $p'$ is such that $1/p + 1/p' = 1$ (that is, $p' = p/(p-1)$), and $a_1, \ldots, a_N$ and $b_1, \ldots, b_N$ are positive real numbers, then

$$\sum_{n=1}^{N} a_n b_n \leq \left( \sum_{n=1}^{N} a_n^p \right)^{1/p} \left( \sum_{n=1}^{N} b_n^{p'} \right)^{1/p'} . \tag{3}$$

There is strict inequality unless $(a_n^p)$ and $(b_n^{p'})$ are proportional, that is unless $a_n^p / b_n^{p'}$ is independent of $n$.

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**Cauchy-Schwartz inequality**

If $p = p' = 2$, Hölder's inequality becomes Cauchy-Schwartz inequality

$$\left( \sum_{n=1}^{N} a_n b_n \right)^2 \leq \left( \sum_{n=1}^{N} a_n^2 \right) \left( \sum_{n=1}^{N} b_n^2 \right) .$$