<u>MSM2P01</u> SEQUENCES AND SERIES, EXERCISE SHEET 2.

Due by 12:00 Monday 9th February 2009.

(1) (a) Give an example of a convergent sequence of real numbers (a_n) such that the sequence (na_n) is bounded. What is the limit of your sequence (a_n)? Explain your answer.

(b) Now suppose that (a_n) is a sequence of real numbers such that (na_n) is bounded. Prove that $a_n \to 0$ as $n \to \infty$.

(2) Let $a_0 = 1$, $a_1 = 1$, and for $n \in \mathbb{N} = \{0, 1, 2, 3, \dots\}$, let

$$a_{n+2} = \frac{1 + a_n + a_{n+1}}{7}.$$

(a) Show that $a_{n+2} - \frac{1}{5} = \frac{1}{7}\big((a_n - \frac{1}{5}) + (a_{n+1} - \frac{1}{5})\big)$.
Hence using the induction hypothesis,

$$H(n): \qquad \forall k \in \mathbb{N}(k \leq n \Rightarrow a_k > \frac{1}{5})$$

show that $a_n > \frac{1}{5}$ for all n.

(b) Use (a) above and an additional induction on n to show that

$$0 < a_n - \frac{1}{5} < \frac{2}{2^n}$$

for all n.

(c) Since $\frac{1}{2^n} \to 0$ as $n \to \infty$ we have

$$\forall \varepsilon > 0 \qquad \exists N \in \mathbb{N} \qquad \forall n \geq N \Big(\frac{1}{2^n} < \varepsilon\Big)$$

Use this together with (b) to prove directly from the definition of convergence for (a_n) that $a_n \to \frac{1}{5}$ as $n \to \infty$.

(3) Either by quoting theorems on boundedness, subsequences, uniqueness of limits, or anything else from the course so far, or by arguing directly from the definition, prove that the following sequences do not converge to any limit.
$a_n = 2^n$; $b_n = \frac{(-1)^n}{10000000} + \frac{1}{n}$; $c_n = \sin \frac{3n\pi}{4}$; $d_n = \sin^2 n$.

You can find all exercise sheets on
http://web.mat.bham.ac.uk/~malevao/MSM2P01/