PGTC 2019

Contributed talks

Cesare Giulio Ardito (University of Manchester)
Morita equivalence classes of blocks with defect group C_2^5

Donovan's conjecture predicts that given a p-group D there are only finitely many Morita equivalence classes of blocks of group algebras with defect group D. While the conjecture is still unknown for a generic p-group D, the conjecture has been proven in 2014 by Eaton, Kessar, Külshammer and Sambale when D is an elementary abelian 2-group, and in 2018 by Eaton and Livesey when D is any abelian 2-group. The proof, however, does not describe these equivalence classes explicitly. A classification up to Morita equivalence over a complete discrete valuation ring $\mathcal{O}$ has been done, among other examples, for D with rank 3 or less, and for $D = C_2^4$.

I have classified blocks of finite groups with defect group $D = C_2^5$ over an algebraically closed field k, and in this talk I will introduce the topic, give the relevant definitions and describe the process of classifying these blocks.

Chris Attenborough (University of York)
Buildings and relative complete reducibility

Buildings can be thought of as nice geometric objects for groups to act on. The theory of buildings was developed by Jacques Tits in order to better understand the algebraic structure of certain groups. In this talk I will provide a brief introduction to buildings before outlining how recent results concerning relative complete reducibility (a generalization of Serre's notion of G-complete reducibility) may lead to an extension of Tits' centre conjecture.

Matthew Conder (University of Cambridge)
Discrete and free two-generated subgroups of SL_2 and PSL_2

Two-generated subgroups of $\mathrm{SL}_2(\mathbb{R})$ and $\mathrm{PSL}_2(\mathbb{R})$ have been extensively studied in the literature. In particular, there is an algorithm which, given any two elements of $\mathrm{SL}_2(\mathbb{R})$ or $\mathrm{PSL}_2(\mathbb{R})$, will determine after finitely many steps whether or not the subgroup generated by these elements is both discrete and free of rank two. This arises from examining the action of these groups on the hyperbolic plane by Möbius transformations.

Two-generated subgroups of SL_2 and PSL_2 over other fields are not so well studied. In the case of a non-archimedean local field K, such as the p-adic numbers $\mathbb{Q}_p$, the groups $\mathrm{SL}_2(K)$ and $\mathrm{PSL}_2(K)$ act on the corresponding Bruhat-Tits tree. In this talk I will discuss how studying this action yields an algorithm which, given any two elements of $\mathrm{SL}_2(K)$ or $\mathrm{PSL}_2(K)$, determines after finitely many steps whether or not the subgroup generated by these elements is discrete and free of rank two.

Alexandra Embleton (Royal Holloway, University of London)
Cohomological invariants of Bredon modules

We introduce the concept of an orbit category for any given family of subgroups of a group and following this the concept of a Bredon module over the orbit category. This then allows free and projective Bredon modules along with their respective resolutions to be defined. Once these preliminaries have been introduced we compare certain cohomological invariant defined in this context such as $silp_{\mathfrak{F}}G$, $spli_{\mathfrak{F}}G$, $cd_{\mathfrak{F}}G$, $gldim_{\mathfrak{F}}G$ and $fin.dim_{\mathfrak{F}}G$.

Conor Finnegan (University College Dublin)
The Projective Characters of Metacyclic p-Groups

The projective characters of a group can provide us with important information regarding the structure and properties of the group. In this talk, I will give a brief overview of some of the fundamental concepts in projective representation theory, assuming some prior knowledge of ordinary representation theory. I will then discuss how these ideas can be applied in order to find the projective character tables of metacyclic p-groups of positive type.

Saul Freedman (University of St Andrews)
Non-commuting, non-generating graphs of finite groups

Given a group G, we can construct associated graphs that encode certain relations between the elements of G. A well-known example is the generating graph of G, whose vertices are the nontrivial elements of G, with two vertices joined if the elements form a generating set for G. Breuer, Guralnick and Kantor showed in 2008 that if G is a finite non-abelian simple group, then this graph is connected with diameter 2.

We explore, for various families of finite groups, the connectedness and diameter of a related graph, obtained by taking the complement of the generating graph and then removing edges between elements that commute. In many cases, this is achieved by studying the maximal subgroup structures of the relevant groups.

Daniele Garzoni (Università degli studi di Padova)
Minimal invariable generating sets

A subset X of a group G invariably generates G if, when each element of X is replaced by an arbitrary conjugate, the resulting set generates G. This concept was introduced by Dixon in the early nineties with motivations coming from computational Galois theory.

In this talk I will present some results concerning the behaviour of "minimal" invariable generating sets, *i.e.*, sets X which invariably generate a group G, but which do not invariably generate anymore whenever any element is removed from them. Everything will be about finite groups.

Some of the results have been obtained in joint work with Andrea Lucchini; the others are a work in progress with Nick Gill.

Jonathan Gruber (École polytechnique fédérale de Lausanne)
Maximal vectors in tensor products of simple modules for simple algebraic groups and complete reducibility

Let G be a simply connected simple algebraic group over an algebraically closed field k of characteristic $p > 0$. We study tensor products of simple G-modules. Given a maximal vector of weight δ in such a tensor product, we show how to find maximal vectors of weight $\delta' > \delta$ using the hyperalgebra of G. As a consequence, we prove that if the tensor product of two simple p-restricted $\mathrm{SL}_n(k)$-modules is completely reducible than all of its composition factors are p-restricted. Using this, we can reduce the question of complete reducibility of tensor products of simple $\mathrm{SL}_n(k)$-modules to the case where both modules have p-restricted highest weight.

Sam Hughes (University of Southampton)
The cohomology of cocompact Fuchsian groups

Fuchsian groups, or discrete subgroups of the isometry group of the hyperbolic plane $\mathrm{PSL}_2(\mathbb{R})$, are a widely studied class of examples in geometric group theory. The groups have connections to numerous topics in analysis, geometry and number theory. In this talk we will give a brief introduction to Fuchsian groups and their signatures. Assuming some background from homological algebra, we will apply a G-equivariant spectral sequence argument to calculate their integral cohomology groups, recovering a result of Majumdar.

Liam Jolliffe (University of Cambridge)
Schaper layers of Specht modules

Over a field of characteristic zero the Specht modules give a complete set of irreducible representations of the symmetric group, however these are not necessarily irreducible over fields of positive characteristic. There is currently no known way of determining the multiplicities of the irreducible composition factors of these Specht modules in this case. The best tool we have to this end is the a sum formula, analogous to the famous Jantzen sum formula, which first appeared in the thesis of Schaper. This sum formula gives an upper bound on the composition multiplicities, as well as determining precisely which multiplicities are zero. The effectiveness of this formula can be improved by studying the filtration of the Specht module from which the sum arises. In this talk we will try to better understand the layers of this filtration and see how this may improve the upper bound given by the formula.

Julian Kaspczyk (University of Aberdeen)
On finite groups realizing the 2-fusion system of $\mathrm{PSL}(n, q)$

My talk is devoted to the study of finite groups whose 2-fusion system is isomorphic to the 2-fusion system of $\mathrm{PSL}(n, q)$ for some $n > 5$ and some odd prime power q. In particular we will have a look at some results about the centralizers of involutions in such groups. These results form a first step towards a characterization of $\mathrm{PSL}(n, q)$ by its 2-fusion system. The presented work is inteded to be a contribution to Aschbacher's programme which aims at a new proof of the classification of finite simple groups using the theory of fusion systems.

Veronica Kelsey (University of St Andrews)
The generating graph and maximal cocliques

Suppose a group G is 2-generated. Then the generating graph of G is the graph whose vertices are the non-identity elements of G, and with two elements joined by an edge whenever they generate G. A set of vertices of a graph are coclique if no two are joined by an edge. We call a coclique maximal if it is not contained in any larger coclique. In this talk, we will see examples and non-examples of maximal subgroups as maximal cocliques in the group's generating graph.

Marvin Krings (RWTH Aachen University)
Orders of sylow subgroups in primitive permutation groups

Primitive permutation groups are fundamental building blocks in the sense that every finite permutation group can be built from the primitive ones. Apart from the alternating group A_n and the symmetric group S_n of degree n, the primitive subgroups G of S_n are small. For example, in 1980 Praeger and Saxl showed that $|G| \leq 4^n$, which is much smaller than $\frac{n!}{2}$. Since this time, powerful results such as the O'Nan–Scott Theorem, which classifies the primitive permutation groups, and the Classification of the Finite Simple Groups, have become available.

We will bound the order of a Sylow p-subgroup $|P|$ of $|G|$ for some prime p. This order is the largest p-power $p^{\nu_p(G)}$ that divides $|G|$. The bound $|G| \leq 4^n$ implies $\nu_p(G) \leq n \log_p(4)$. We get a stronger bound $\nu_p(G) \leq c \log_p n$ for some constant c unless G belongs to a specific class of groups we call giants. But even for the giants, we can still prove $\nu_p(G) \leq \frac{2\sqrt{n}}{(p-1)} + 1$ (with five exceptions).

Our proof works by considering the cases of the O'Nan–Scott Theorem separately. In this talk, I will give an overview of the techniques we use for each of theses cases.

Alex Levine (Heriot-Watt University)
Solving equations in nilpotent groups

Equations in groups have existed for over a thousand years, even if they were not formalised as equations in groups. For example, linear diophantine equations are just equations over the integers as a group. We explore equations in nilpotent groups of class 2, in particular the Heisenberg group, and describe their relation to quadratic diophantine equations.

Oliver Matheau-Raven (University of York)
Cutoff for a one-sided transposition shuffle

The random transposition shuffle is defined by our hands independently choosing a card each to transpose every step. The one-sided transposition shuffle follows from the modification of our left hand being dependent on the right hand, and not able to cross each other. We show how shuffling a deck of cards may be viewed as a random walk on the symmetric group and give a description of how algebraic techniques can be used to analyse the speed at which this random walk converges to the uniform distribution. We uncovered a remarkable branching structure for the eigenspaces of the one-sided transposition shuffle involving Young diagrams which allows us to label the eigenvalues for this shuffle. After analysis of the eigenvalues we find the number of shuffles required to get close to uniform is $n \log n + cn$. With a matching lower bound found we prove that the one-sided transposition shuffle exhibits a cutoff at time $n \log n$.

Eoghan McDowell (Royal Holloway, University of London)
Clebsch–Gordan rule for $\mathrm{SL}_2(\mathbb{F}_p)$ in characteristic p

A "Clebsch–Gordan rule" for a particular group describes the decomposition of tensor products of its simple representations into indecomposable representations. The rule for $\mathrm{SL}_2(\mathbb{F}_p)$ in characteristic p can be obtained inductively by tensoring simple modules and their projective covers with the natural module. In this talk I will define a new family of maps between tensor products of representations of $\mathrm{SL}_2(\mathbb{F}_p)$ and show how this leads to a more economical proof of its Clebsch–Gordan rule in characteristic p. I will also demonstrate an intuitive way to visualise these decompositions.

Eilidh McKemmie (University of Southern California)
Invariable generation of finite classical groups

We say a group is invariably generated by a set if any conjugates of those elements generates the group. Eberhard, Ford and Green built upon the work of many others and showed that, as $n \to \infty$, the probability that S_n is invariably generated by a random set of elements is positive if there are four random elements, but goes to zero if we pick three random elements. This result gives rise to a nice Monte Carlo algorithm for computing Galois groups of polynomials. We will extend this result for S_n to the finite classical groups using the correspondence between classes of maximal tori of classical groups and conjugacy classes of their Weyl groups.

Saadia Mehwish (The Islamia University of Bahawalpur)
Graham Higman's question regarding Januarials

Januarials were defined by Graham Higman in his last series of lectures. In this talk I will answer some questions posed by Higman in these lectures (Stated: For a given k, what are the possible values for and interrelationships between h, g1 and g2 for januarials of simple type? And a similar question for januarial of general type. Are there arbitrarily large values of k for which there exist simple januarials with only one simple disjoint circuit, that is, h=1?)

Carmine Monetta (Università degli Studi di Salerno)
Boundedly finite conjugacy classes of tensors

The non-abelian tensor square $G \otimes G$ of a group G as introduced by Brown and Loday is defined to be the group generated by all symbols (called tensors) $g \otimes h$, with g, $h \in G$, subject to the relations

$$gg_1 \otimes h = (g^{g_1} \otimes h^{g_1})(g_1 \otimes h) \quad \text{and} \quad g \otimes hh_1 = (g \otimes h_1)(g^{h_1} \otimes h^{h_1})$$

for all g, g_1, h, $h_1 \in G$. Hence the defining relations of the tensor square can be viewed as abstractions of commutator relations. The set of all tensors, denoted by $T_{\otimes}(G)$, deeply affects the structure of the non-abelian tensor square. For example, Bastos, Nakaoka and Rocco proved that $G \otimes G$ is finite if and only if $T_{\otimes}(G)$ is finite.

We say that a group G is a BFC-group if there exists a positive integer n such that the conjugacy class $x^G = \{x^g \mid g \in G\}$ of every element $x \in G$ has size at most n. Schur proved that if G is a central-by-finite group, then the derived subgroup G' is finite, and the group G is a BFC-group. Neumann improved Schur's theorem, showing that a group G is a BFC-group if and only if the derived subgroup G' is finite. In this direction, Dierings and Shumyatsky proved that if $|x^G| \leq n$ for every commutator x in G, then the second derived subgroup G'' is finite with n-bounded order.

The aim of this talk is to present similar results in the context of non-abelian tensor square, obtained in collaboration with Bastos.

Adán Mordcovich (University of St Andrews)
On maximal non-trivial subgroups of almost simple groups

We know what the largest maximal subgroups of simple groups are so a natural extension would be to ask the maximal subgroups of *almost simple groups* are. An almost simple group here is one that satisfies $S \leq G \leq \mathrm{Aut}(S)$ for some simple non-abelian group S. We would expect in these cases that information regarding maximal subgroups of S could carry on over to the G themselves, however simply asking what the largest subgroups of G usually eliminates a lot of this information (The largest subgroup here would most likely contain the whole of S itself.). So we tackle the question of what the largest *non-trivial* maximal subgroups of G are, *non-trivial* here means not containing S. In this talk we aim to discuss how the subgroups of S relate to the non-trivial subgroups of G, what the largest non-trivial maximal subgroups of G are, and if time permits, some possible applications of this.

Mariapia Moscatiello (Università degli Studi di Padova)
The expected number of random elements to generate a finite group

A classical result in the context of generation of finite groups, proved in 1989 separately by R. Guralnick and A. Lucchini, states that if all the Sylow subgroups of a finite group G can be generated by d elements, then the group itself can be generated by $d + 1$ elements.

We will talk about a probabilistic version of this result. Let G be a finite group, denote by $e(G)$ the expected number of elements of G which have to be drawn at random, with replacement, before a set of generators is found. If all the Sylow subgroups of G can be generated by d elements, then $e(G) \leq d + \kappa$ where κ is an absolute constant that is explicitly described in terms of the Riemann zeta function and best possible in this context. Approximately, κ equals 2.752394.

We will discuss about a generalization of the previous result for finite soluble groups. If G is a soluble group having, for every $p \in \pi(G)$, a subgroup G_p such that p does not divide $[G : G_p]$ and $e(G_p) \leq \rho$, then $e(G) \leq \rho + 9$.

Sam Mutter (Newcastle University)
Counting bi-Eulerian paths around regular multigraphs

It is well-known that the modular group $M \coloneqq \mathrm{PSL}(2, \mathbb{Z})$ can be characterised as the free product of cyclic groups of order 2 and 3. Using this fact, we can study coset diagrams of the Cayley graph of M in order to count subgroups of index g in M. We make use of words called Wicks forms to count these graphs, and explore ways of generalising this method to counting subgroups of Hecke groups.

Robert Nicolaides (Univeristy of Manchester)
Abstract Regular Polytopes and The Existence of C-string Groups

Abstract Regular Polytopes can be thought of as a modern generalisation of the Convex Regular Polytopes including the famed Platonic Solids. We will introduce the concept of an Abstract Regular Polytope intuitively and state their correspondence theorem with String C-Groups. Then we will consider some specific groups and ask if there exists some abstract regular polytopes with them as their Automorphism Group.

Aura-Cristiana Radu (Newcastle University)
Projective covers and calculating extensions of simple modules

Let G be a finite group over F an algebraically closed field of characteristic p and let FG be its group algebra. Let M, N be 2 simple FG-modules. We want to find all possible FG-modules E such that

$$0 \to M \to E \to N \to 0$$

is a short exact sequence.

There exists a natural equivalence relation of these SESs, which we denote by $\mathrm{Ext}^1_{FG}(M, N)$. The aim of this talk is to introduce the notion of a projective cover of a module and explain how it can be used to compute extensions of simple modules. I will also present some results in the case of Ree groups of type F_4 in characteristic 2, that I am currently researching.

Aluna Rizzoli (Imperial College London)
Finite singular orbit modules for algebraic groups

Building on the classification of modules for algebraic groups with finitely many orbits on subspaces, we determine all irreducible modules for semisimple algebraic groups that are orthogonal and have finitely many orbits on singular 1-spaces. This question is naturally connected with the problem of finding for which pairs of subgroups H, K of an algebraic group G there are finitely many (H, K)-double cosets. We provide a solution to the question when K is a maximal parabolic subgroup P_1 of a classical group SO_n. We find an interesting range of new examples ranging from a 5-dimensional module for SL_2 to the spin module for B_6 in characteristic 2.

Mehmet Sefa Cihan (University of Essex)
Generalised graph groups with $|V| = |A| - 1$

A class of generalized graph groups with $|V| = |A| - 1$, where $|V|$ is the number of vertices and $|A|$ is the number of edges. The paper will turn its attention to prove whether these corresponding groups of the possible graphs under $|V| = |A| - 1$ are finite cyclic group or not. In some cases we are able to show that they are finite cyclic group (18 of 25 possible graphs), in others at least that rank is 1 or 2. We also demonstrate the order of these groups.

Patrick Serwene (City, University of London)
The Equivalence of exotic and block exotic Fusion Systems

One of the main problems in the theory of fusion systems is the question whether a fusion system arises in the form of a finite group if and only if it arises in the form of a p-block of a finite group. There is a conjecture saying that a fusion system is induced by a group if and only if it is induced by a block. We will present reduction theorems for this question which reduce this problem to blocks of quasisimple groups in certain cases. We will use one of these reductions to settle this question for the family of Parker–Semeraro systems. Finally, we will discuss ongoing work concerning our strategy in order to prove the conjecture for some (simple) groups of Lie type.

Ben Stratford (The University of Warwick)
Classifying the primitive permutation groups of small degree

An introduction to the methods used and problems that occur in the classification of the primitive permutation groups of finite degree. This will include a brief overview of the O'Nan-Scott Theorem and Aschbacher's Theorem, although many details will be omitted.

Vladimir Vankov (University of Southampton)
Special and extraspecial groups

We will learn how certain interesting classes of finite groups can appear in research relating to infinite groups which are not even finitely presented. Motivated by the geometry of branched covers of special cube complexes, we will see how representation theory of finite groups can help to answer questions of geometric group theory concerning generalised Bestvina-Brady groups and finiteness properties.