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Subcontinua of $\beta\mathbb{R}$

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We show that the Cech-Stone remainder of the real line has the maximum possible number of mutually non-homeomorphic subcontinua, to wit 2^c

Set Functions and Irreducible Continua

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A *continuum* is a non-empty compact connected metric space. Given a continuum X, we define the hyperspaces:

$$2^X = \{A \subseteq X : A \text{ is closed and non-empty}\}$$

and

$$C(X) = \{A \in 2^X : A \text{ is connected}\}$$

The hyperspace 2^X is known as *the hyperspace of closed and non-empty subsets of* X and $C(X)$ is *the hyperspace of subcontinua of* X.

The *set functions* $\mathcal{T}$ and $\mathcal{K}$ are defined as follows: for each $A \subseteq X$,

$$\mathcal{T}(A) = X \setminus \{x \in X : \text{ there is } W \in C(X) \text{ such that } x \in Int(W) \subseteq W \subseteq X \setminus A\}$$

and

$$\mathcal{K}(A) = \cap \{W \in C(X) : A \subseteq Int(W)\}$$

A continuum X is *irreducible* if there exist two points of X such that no proper subcontinuum of X contains both points. Some properties of the set functions $\mathcal{T}$ and $\mathcal{K}$ on irreducible continua are going to be presented.

The dimension of a generalised inverse limit

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If for each $i \in \mathbb{N}$, $f_{i+1} : [0,1] \to 2^{[0,1]}$ is an upper semicontinuous function, then for each $n \geq 1$ let $\mathcal{G}(f_1, \ldots f_n) = \{\langle x_0, \ldots, x_n \rangle \in [0,1]^{n+1} : \forall\, i < n, x_i \in f_{i+1}(x_{i+1})\}$. Given any pair of positive integers m, n, we give conditions on the graphs of bonding functions that must be satisfied if and only if $\mathcal{G}(f_1, \ldots f_n)$ contains an m-cell.

Dynamics of generalized inverse limits over intervals

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Abstract: Suppose $f : I \to 2^I$ is a surjective, upper semicontinuous bonding map. Let M denote the generalized inverse limit space generated by f. Even though f is not even a function in the usual sense, it induces a continuous function σ from M onto M. The function σ is called the shift map on M, since for $\mathbf{x} = (x_0, x_1, ...) \in M$, $\sigma(\mathbf{x}) = \sigma(x_0, x_1, ...) = (x_1, x_2, ...)$.

We begin an investigation of the dynamical behavior of the shift map σ. We would like to know which properties of the bonding map f imply that the map σ has certain dynamical properties. So far, we have considered properties which imply the following:

1. The points $\mathbf{0} := (0, 0, 0, ...)$ and $\mathbf{1} := (1, 1, 1, ...)$ are fixed under σ, and the point $\mathbf{0}$ attracts all points of M except $\mathbf{1}$; i.e., if $\mathbf{x} \in M$, $\mathbf{x} \neq \mathbf{1}$, then $\lim_{n\to\infty} \sigma^n(\mathbf{x}) = \mathbf{0}$.

2. The shift map σ is topologically transitive on M.

3. The shift map σ has the property that every (nonempty) open set $U \subset M$ contains a periodic nonempty closed set.

4. The inverse limit space M admits a Cantor set $\mathcal{A}$ of nonempty closed sets invariant under the shift map σ. Furthermore, if σ' represents the continuous mapping on the hyperspace 2^M of M induced by σ on M, then $\sigma' |_{\mathcal{A}} : \mathcal{A} \to \mathcal{A}$ is a shift on n symbols.

5. The shift map σ has positive topological entropy.

This is just the beginning.

Dynamical properties of the doubling map with holes

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We study the attractors of the family of open dynamical systems (colloquially, maps with holes) corresponding to the doubling map. We analyse some of their properties such as transitivity and the specification property when the open dynamical system corresponds to a centred symmetric hole. If time permits, some partial results on the asymmetric case will be mentioned.

Ramsey shadowing and recurrence

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We say that a dynamical system X has *Ramsey shadowing* if an arbitrary sequence of points in X can be ε-shadowed, for any $\varepsilon > 0$, on a set that is “large” in the sense of Ramsey theory. We will show that this property is actually a kind of recurrence property. Specifically, we will define two similar-seeming recurrence properties and show that Ramsey shadowing is implied by one and implies the other. Furthermore, we will show that in the presence of shadowing the Ramsey shadowing property is equivalent to chain recurrence.

Shadowing in Dynamical Systems

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A continuous map on a compact metric space has the shadowing property provided that for all $\epsilon > 0$ there exists a $\delta > 0$ such that each δ-pseudo-orbit is ϵ-shadowed by a true orbit of the map.

We will discuss some recent results about the shadowing property in several contexts.

Asymptotic group actions and their limit
"Topology mopping up missed items in classical analysis"

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Inspired by joint work with: N.H. Bingham, Imperial College London

Classical (Karamata) theory, redefined to the topological context, with a group A acting on a group G, characterizes the 'regularly varying' functions $f : G \to H$ between metrizable topological groups, i.e. those that possesses a well-defined limit $\partial_A f(t) := \lim_x f(tx)/f(x)$, (e.g. when the groups are locally compact, or when the right-invariant metric diverges to infinity: $||x|| := d_R^G(x, 1_G) \to \infty$), as approximate homomorphisms.

Beurling's notion (from 1957) of 'slow variation' (introducing the more general *Beurling convolution* to significantly extend Wiener's Tauberian Theorem) has only recently been extended to an analogous theory of Beurling regular variation, but so far only in the context of $\mathbb{R}$.

The recent algebraicization of Beurling's convolution (associated with the Beurling transformations $t : x \to x + t\varphi(x)$ with $t \in \mathbb{R}$ and φ a *self-equivarying flow*) draws on the Popa-Javor group theory of $\mathbb{R}$.

The latter opens the door to a topological analogue, which requires the unusual notion of 'aymptotic group action' – a 'Lie-like' structure – within which limit groups of Popa type arise. In this context functional equations that extend the Cauchy equation all reduce to homomorphisms.

An internal characterisation of radiality

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Introduced by Horst Herrlich in 1967 [Her67], radial spaces are a generalisation of Frchet-Urysohn spaces and can be characterised as the pseudo-open images of either spaces with well-ordered neighbourhood bases (well-based spaces) or LOTS. In this talk, I will present results from [Lee1X] that arose from considering what makes a space radial. By abstracting the characteristics of LOTS that make them radial, a new class of spaces is found that encapsulates both GO- and well-based spaces, and is strictly contained in the class of radial spaces. Weakening the definition provides an exact internal, local characterisation of radiality.

References: [Her67] Horst Herrlich. Quotienten geordneter Rume und Folgenkonvergenz. Fund. Math., 61:7981, 1967. [Lee1X] R. Leek. An internal characterisation of radiality. Topology Appl., forthcoming. Pre-print available at http://arxiv.org/abs/1401.6519

ZFC results at singular cardinals

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Singular cardinals, as the name suggests, have traditionally been considered as the weird place where reasonable arguments break, starting from J. Knigs famous failed proof of the non-well order ability of the reals in 1904. The more recent experience shows however that it is exactly at the singular cardinals where ZFC is most powerful, and in particular that some unexpected theorems hold there. We shall discuss some celebrated results at singular cardinals and two recent theorems and then make a call for a more systematic investigation of singulars, not least in topology.

The Stone-Cech remainder of $\omega^* \setminus \{x\}$

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Fine and Gillman showed in 1960 that under CH any space of the form ω^* minus a point has a non-trivial Stone-Cech remainder. In this talk we investigate topological properties of these remainders and show that even though the spaces $\omega^* \setminus \{x\}$ are typically very different, it is consistent with CH that all their remainders are homeomorphic.

Topological interpretations of the gap free betweenness axiom

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An interpretation of betweenness on a set is gap free if each two distinct points of the set have a third point between them. In this talk we are interested in gap free betweenness relations naturally arising in connected topological spaces.

Generating Topologies from Nests

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In 1972, J. van Dalen and E. Wattel provided a characterisation of linearly ordered topological spaces as those which can be generated by a pair of interlocking nests. We extend these ideas to more than two nests.

Completeness properties of initial topologies

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We consider initial topologies generated by rings of functions with compact image. We show that such an initial topology need not be pseudocompact, but is Baire.

Compactifications and representability of groups

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Given a topological group G a semigroup compactification of G is a pair (S, T) where S is compact right topological semigroup (right topological meaning that the right translation map $s \to st$ is continuous for every t) and $T : G \to S$ is a continuous homomorphisms with dense image and such that, when t is in $T(G)$, the left translation by t is a continuous mapping.

In this talk we shall be interested in universal semigroup compactifications arising from admissible algebras of functions. Namely, we shall concentrate on the Almost Periodic (AP) compactification and the Weakly Almost Periodic (WAP) compactification.

The aspect of the theory we shall present is the correspondence there is between the "size" (for lack of a better term) of these compacifications, the richness of the corresponding algebras of functions and the possibility of finding representations of the original group into a Hilbert Space and into a Reflexive Banach Space.

Once presented (very briefly) the general theory we shall concentrate on the WAP-compactification and the corresponding property of being reflexively representable.

In particular we shall show some results holding for metric groups which relate a specific property of the metric with reflexive representability.

The talk is meant to be a presentation of the general theory more than an account of a specific result, however I shall present also some original result which have been obtained in a joint work with Jorge Galindo and a joint work with Itaï Ben Yaakov and Alexander Berenstein.

On $\mathcal{I}^{\mathcal{K}}$- Cauchy functions

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In this paper we introduce the notion of I^K-Cauchy function, where I and K are ideals on the same set. The I^K-Cauchy functions are a generalization of I^*-Cauchy sequences and I^*-Cauchy nets. We show how this notion can be used to characterize complete uniform spaces and we study how I^K-Cauchy functions and I-Cauchy functions are related.

Matchbox Manifolds: their structure and classification

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A matchbox manifold is a continuum that locally is homeomorphic to the product of an open disk of some fixed dimension and a totally disconnected space. In this talk we will give an overview of some recent results on the structure and classification of these spaces.

Open tilings of topological spaces

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The goal is to develop a concept of "tiling" a given topological space with tiles of richer structure. A space has an open tiling (OT) if for each open set U, there exists a collection T of mutually disjoint open sets such that the closure of each member of T is contained in U, and U is equal to the union of the closures of the members of T. An alpha-open tiling (alpha-OT) of a space is a an OT in which each open set has a tiling of cardinality less than or equal to alpha. In this talk we restrict our concern to regular, Hausdorff, first countable spaces, and consider which spaces have an OT or omega-OT. We also consider which non-Moore spaces can be tiled by Moore tiles, and which non-metrisable spaces can be tiled by metrisable tiles. A few basic theorems are given. Non-Moore examples such as (1) omega-1 with the order topology, (2) the unit square with the lexigraphic order, (3) the Michael line, (4) the Sorgenfrey line, (5) McAuley's butterfly space, and (6) the Ostaszewski line are examined, as are Moore non-metrisable spaces such as (1) the tangent disk spce (including the version over a Q-set and the version with points in the upper plane isolated), (2) the space psi, (3) Heath's V-space, (4) the Pixley-Roy space, (5) the author's examples of (i) a Moore space with a sigma-disjoint pi-base but with no sigma-discrete pi-base, (ii) countably paracompact Mooore space which are not normal, and (iii) collectionwise Hausdorff Moore spaces which are not normal. The results are surprising, at least to the author. In classical Moore tradition, the author suggests that readers should find their own solutions. It is fun.