Preface

How wonderful that we have met with a paradox.
Now we have some hope of making progress.
Niels Bohr

Capillarity plays an important role in numerous natural phenomena and various technological processes. The latter range from film flows in coating devices and dynamics of drops in ink-jet printing to biological and biomedical applications of fluid mechanics, bioengineering and microfluidics. The complexity of effects associated with capillarity and a continuously widening spectrum of applications have kept capillary flows in the focus of considerable research effort for more than two centuries. This effort brought in many remarkable successes in the understanding of the fundamentals and the quantitative modelling of capillary phenomena in different situations.

However, research also highlighted a number of intriguing paradoxes where an attempt to describe mathematically some, often quite ordinary, capillary flows by combining well-tested elements from the arsenal of classical fluid mechanics leads to solutions with manifestly unphysical properties. Furthermore, in some situations there are no solutions at all! These 'paradoxical flows' include the spreading of liquids on solid surfaces, coalescence and breakup of drops and bubbles, breakup of jets, disintegration of liquid films on a solid substrate, and the formation of two-dimensional singularities in the free-surface curvature, to mention but a few examples.

In each case when a paradox is encountered, it becomes a *problem*. Such *problems* mark the boundary that the standard computer-aided applied research cannot cross without descending to a semi-empirical level. Ironically, many applications, especially in the emerging technologies, lie on the other side of this boundary and cannot be 'scaled up' to be dealt with on a semi-empirical basis. As a result of this situation, each of these *problems* becomes the subject of intensive research in its own right, with its own 'club' of investigators, ad hoc 'laws' and all other attributes of a long siege.

The key idea of the present monograph is a realization that many seemingly very different 'paradoxical flows' are in fact particular cases from a general class of fluid motion — capillary flows with *formation* (and/or disappearance) of interfaces. For example, the famous 'moving contact-line problem' arises in the continuum description of the process known as 'dynamic wetting', and the very name 'dynamic wetting' actually means a process by which an initially dry solid surface becomes 'wet', i.e., a process of *formation* of a new liquid-solid interface (a 'freshly wetted' solid surface). Then it becomes clear why classical fluid mechanics fails to adequately describe dynamic wetting as well as other flows where the formation of interfaces is essential: this process is simply not accounted for in the model. It is also clear why treating 'paradoxical flows' separately on an ad hoc basic, as purely mathematical *problems*, leads to less than satisfactory results: there is very little chance that one will accidentally come up with an adequate mathematical description of a phenomenon without realizing what physics controls it and the place of this physics in a broader physical context.

The objectives of this book are described in a section of the first chapter entitled "Scope of the book". Here it is appropriate to outline the book's style, methodology and structure.

In this respect, the main overall goal was to achieve textbook clarity in the exposition of both the physical ideas and the mathematics they are wrapped in. To bring the readers with different educational backgrounds to the same starting level, assumed to be the level of the final-year undergraduate students, the first two chapters give a research-oriented exposition of the fundamentals of the fluid mechanics. This part can be seen as a concise 'textbook for future researchers'. A particular emphasis is on the topics that are rarely covered in standard texts, like the Lagrangian representation of the equations of motion, the use of the Airy stress function for the Stokes flows, assumptions behind boundary conditions, especially on the solid surface, and their limits of applicability. To make the reading easier, appendices recapitulate the necessary elements of mathematics in a self-sufficient user-friendly way. Since the clause "as is known" often becomes a barrier separating knowledge from familiarity, it is completely avoided in the first two chapters and wherever possible in what follows.[1] Chapter 2 ends with a discussion of basic criteria that must be satisfied by a physically meaningful solution of a mathematical problem formulated in the framework of fluid mechanics. These minimal criteria of physical meaningfulness and self-consistency can then be used to assess theories proposed to remove the paradoxes.

The subsequent chapters are devoted to the 'paradoxical flows'. In each case, first, we discuss the essence of the problem inherent in the classical fluid mechanics formulation. This is followed by a detailed review and analysis of relevant experiments and, where available, theories developed alongside them to remedy the problem. In this analysis, the experiments are looked at through the eyes of a theoretician and the conceptual frame of fluid mechanics since, as Einstein put it, "it is the theory that decides what we can observe". This remark is particularly relevant to the phenomena that we consider.

[1] Aristotle's observation that "what is known is known to a few" seems to be as true now as it was in his time.

The theories are examined against the following:

(i) The basic criteria of physical relevance,

(ii) The underlying assumptions and hence the limits of applicability of continuum mechanics within which these theories have been formulated as well as assumptions behind the theories themselves,

(iii) Their ability to describe experimental observations and provide a conceptual framework which would allow one, if necessary, to incorporate other physical effects.

This analysis allows the reader to have a broad picture of the situation with the flow in question and see advantages as well as disadvantages of different theoretical approaches to its modelling.

Then, we consider how the common root of the problems with 'paradoxical flows' — the missing physics of the interface formation in the mathematical model — emerges from different guises. Once the essence of the difficulty is identified for the first time (Chapter 3), we develop the simplest (irreducible) ready-to-use model addressing the difficulty (Chapter 4) which we then test, without any ad hoc adjustments, for other flows against the *same criteria* as the theories reviewed earlier. This approach ensures consistency of the analysis and indicates how the model's parameters can be determined from independent experiments.

The book is intended for the following categories of readers:

- Graduate and final-year undergraduate students in applied mathematics, physics and engineering whose programs of research or study includes fluid mechanics,
- Chemical, mechanical, design and bioengineers working with fluids, in particular, in the areas of microfluidics and coating technologies,
- Researchers in applied mathematics and physics interested in capillary flows, interfacial phenomena and related aspects of fluid mechanics.

Throughout the book cross-referencing is used to highlight logical connections between different phenomena and the unified way in which they are addressed. In order to counterbalance this interweaving of material, the text is punctuated with summaries that outline the main points and are intended to facilitate practical use of the book. More than 130 figures provide visual support and illustration of the main concepts. The author-plus-year referencing system together with the Index of Authors make it easy to find references to particular works. The references include all main publications directly related to the problems considered and the physics of interface formation; references to the works that deal primarily with influences *additional* to the physics of interface formation (effects of electromagnetic field, surfactants, chemical reactions, evaporation/condensation, etc.) have not been included and can be found in the cited papers and reviews.

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