# MSM2P01, Autumn 2013, Exercises 1

**Instructions.** Read and attempt all questions before the examples class on Tuesday. You will have some time in the class to ask questions for clarification or hints, but if you do not prepare for the class you will not be able to get the most out of it. Write up your own work (on your own!) after the class and hand it in to the white pigeonhole for this module by 1300 on the Friday of the same week as the examples class.

**Note.** The mathematics in the following exercises concerns straightforward inequalities, so in this respect the mathematics here work should be revision. However the exercises also develop mathematical language, logic and rigour too. You are also being tested on this.

You should *only* answer the questions that are asked. If you do more work than is required we will know that you misunderstood the question and you may have marks deducted.

It should also be understood throughout this module that **any assertion you make as part of any answer *must* be justified by a reason or proof**, irrespective of whether the question asks for one. This is from now on implicit in every question at all times. We will regard an unjustified assertion as automatically 'incorrect'.

For this sheet you will need the following definition.

**Definition 1.** The function $|\cdot|\colon \mathbb{R} \to \mathbb{R}$ is defined by $|x| = x$ for $x \geqslant 0$ and $|x| = -x$ for $x < 0$.

**Exercise 2.** (a) Find the set of all $x \in \mathbb{R} \setminus \{-3, -5\}$ such that $\frac{x+1}{x+3} < \frac{x+3}{x+5}$.

(b) Find the set of all $x \in \mathbb{R} \setminus \{-3, -4\}$ such that $\frac{x+1}{x+3} < \frac{x+3}{x+4}$.

(c) Find the set of all $x \in \mathbb{R} \setminus \{-3, -2\}$ such that $\frac{x+1}{x+3} < \frac{x+3}{x+2}$.

Give your answers as an interval or a union of disjoint intervals.

**Exercise 3.** Let $a_n = \dfrac{9n+1}{(3n+1)(n+8)}$.

(a) Find $N \in \mathbb{N}$ such that for all integers $n > N$ it is true that $|a_n| < 1/100$.

(b) Find $N \in \mathbb{N}$ such that for all integers $n > N$ it is true that $|a_n| < 1/100000$.

(c) Find $N(\varepsilon)$ (a function of one positive real variable taking natural number values, i.e. $N\colon \mathbb{R}^+ \to \mathbb{N}$) such that for all real $\varepsilon > 0$ and all integer $n > N(\varepsilon)$ it is true that $|a_n| < \varepsilon$.

**Definition 4.** We say that '$a_n \to l$ as $n \to \infty$' when there is $N(\varepsilon)$ (i.e. a function $N\colon \mathbb{R}^+ \to \mathbb{N}$) such that

For all real $\varepsilon > 0$ and all integer $n > N(\varepsilon)$ it is true that $|a_n - l| < \varepsilon$.

**Exercise 5.** Let $a_n = (-1)^n$ and let $l \in \mathbb{R}$ be a real number.

(a) Write down a statement that is equivalent to '"$a_n \to l$ as $n \to \infty$" is false'.

(b) Find some real $\varepsilon > 0$ (which may or may not depend on $l$) such that '$|1 - l| \geqslant \varepsilon$ or $|-1 - l| \geqslant \varepsilon$' is true.

(c) Prove your statement in (a).

## Ex1: Answers, commentary and feedback