More on κ-like models

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In this document, I will list any extra observations since writing my two papers on κ-like models [1, 2]. The notation follows that in those papers.

Theorem 1 $\mathrm{IPHP} \vdash \mathrm{IPHP}_{\mathrm{cf}}$

Proof. Let $M \models \mathrm{IPHP}$. Then either $M \models \mathrm{PA}$ in which case $M \models \mathrm{IPHP}_{\mathrm{cf}}$, or $M \models \neg B\Sigma_n$ for some n. Suppose $b \in M$ and

$$M \models \forall x{<}a\, \exists y\, \theta(x,y) \wedge \forall z\, \exists x{<}a\, \forall y{<}z\, \neg\theta(x,y).$$

Let $K \succ M$ be a proper extenstion, where the extension preseves $\{x : x < \max(a,b)\}$; K exists by IPHP. It suffices to show that $M \subseteq_{\mathrm{cf}} K$; but if not there is $c \in K$ greater than all $y \in M$, so by elementarity

$$K \models \forall x{<}a\, \exists y{<}c\, \theta(x,y)$$

hence

$$M \models \exists z\, \forall x{<}a\, \exists y{<}z\, \theta(x,y)$$

a contradiction.

References

[1] R. W. Kaye, 'Constructing κ-like models of arithmetic', to appear in *The Journal of the London Mathematical Society*, 1996.

[2] R. W. Kaye, 'The theory of κ-like models of arithmetic', *Notre Dame Journal of Formal Logic* **36**, Fall 1995, 547–559.