Example 4.13, page 67.

Replace this with the following.

Example 4.13 In the complex vector space $\mathbb{C}^2$, take ordered bases $\mathbf{a}_1, \mathbf{a}_2$ and $\mathbf{b}_1, \mathbf{b}_2$ where

$$\mathbf{a}_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}, \ \mathbf{a}_2 = \begin{pmatrix} 1 \\ 1+i \end{pmatrix}, \qquad \mathbf{b}_1 = \begin{pmatrix} i \\ -i \end{pmatrix}, \ \mathbf{b}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Then the base-change matrices from the usual basis $(1,0)^T, (0,1)^T$ to $\mathbf{a}_1, \mathbf{a}_2$ is

$$\mathbf{P} = \begin{pmatrix} 1 & 1 \\ i & 1+i \end{pmatrix},$$

and from the usual basis $(1,0)^T, (0,1)^T$ to $\mathbf{b}_1, \mathbf{b}_2$ is

$$\mathbf{Q} = \begin{pmatrix} i & 1 \\ -i & 1 \end{pmatrix}.$$

This can be used to find the base-change matrix from $\mathbf{a}_1, \mathbf{a}_2$ to $\mathbf{b}_1, \mathbf{b}_2$ as follows.

If $(v_1, v_2)^T$ is the coordinate form of a vector $\mathbf{v}$ with respect to $\mathbf{b}_1, \mathbf{b}_2$, then $\mathbf{Q}(v_1, v_2)^T$ is the coordinate form of $\mathbf{v}$ with respect to the usual basis. Then $\mathbf{P}^{-1}\mathbf{Q}(v_1, v_2)^T$ is the coordinate form of $\mathbf{v}$ with respect to $\mathbf{a}_1, \mathbf{a}_2$, so the base-change matrix from $\mathbf{a}_1, \mathbf{a}_2$ to $\mathbf{b}_1, \mathbf{b}_2$ is

$$\begin{aligned} \mathbf{P}^{-1}\mathbf{Q} &= \begin{pmatrix} 1 & 1 \\ i & 1+i \end{pmatrix}^{-1} \begin{pmatrix} i & 1 \\ -i & 1 \end{pmatrix} = \begin{pmatrix} 1+i & -1 \\ -i & 1 \end{pmatrix} \begin{pmatrix} i & 1 \\ -i & 1 \end{pmatrix} \\ &= \begin{pmatrix} -1+2i & i \\ 1-i & 1-i \end{pmatrix}. \end{aligned}$$