# British Early Career Mathematicians' Colloquium 2020 Abstract Booklet

14th - 15th July 2020

## Plenary Speakers

### Pure Mathematics:

**Jonathan Hickman**
University of Edinburgh

**Liana Yepremyan**
The London School of Economics and Political Science

**Anitha Thillaisundaram**
University of Lincoln

### Applied Mathematics:

**Adam Townsend**
Imperial College London

**Gabriella Mosca**
University of Zürich

**Jaroslav Fowkes**
University of Oxford

### Contact:

Website: http://web.mat.bham.ac.uk/BYMC/BECMC20/
E-mail: becmc2020@contacts.bham.ac.uk

### Organising Committee:

Constantin Bilz, Alexander Brune, Matthew Clowe, Joseph Hyde, Amarja Kathapurkar (Chair), Cara Neal, Euan Smithers.

With special thanks to the University of Birmingham, MAGIC and Olivia Renshaw.

MAGIC

UNIVERSITY OF BIRMINGHAM

## Tuesday 14th July 2020

| | | | |
|---|---|---|---|
| 9.30-9.50 | Welcome session | | |
| 10.00-10.50 | On convergence of Fourier integrals<br>Jonathan Hickman (Plenary Speaker) | | Microscale to macroscale in suspension mechanics<br>Adam Townsend (Plenary Speaker) |
| 10.55-11.30 | Group networking session | | |
| 11.35-12.00 | Strong components of random digraphs from the configuration model: the barely subcritical regime<br>Matthew Coulson | Blocks of finite groups of tame type<br>Norman MacGregor | The evolution of a three dimensional microbubble in non-Newtonian fluid<br>Eoin O'Brien |
| 12.10-12.35 | Large trees in tournaments<br>Alistair Benford | Donovan's conjecture and the classification of blocks<br>Cesare Giulio Ardito | Order from disorder: chaos, turbulence and recurrent flow<br>Edward Redfern |
| Lunch break | | | |
| 14.00-14.50 | Ryser's conjecture and more<br>Liana Yepremyan (Plenary Speaker) | | MorphoMecanX: mixing (plant) biology with physics, mathematics and computer science<br>Gabriella Mosca (Plenary Speaker) |
| 14.55-15.30 | Beginning a Career in Academia (Panel session) | | |
| 15.35-16.00 | Frobenius manifolds in mathematics and beyond<br>Karoline van Gemst | N-centred Lipschitz Quotient Mappings<br>Ricky Hutchins | Layered Bayesian Learning given Highly Discontinuous Data - Application to Materials Science<br>Georgios Stagakis |
| 16.10-16.35 | Describing the associated order in a Hopf Galois structure of extensions of p-adic fields<br>Daniel Gil Munoz | Limit points of Mahler measures coming from digraphs<br>Joshua Coyston | Analysis of temperature and humidity in Oman using modern statistical method<br>Safia Al Marhoobi |
| 16.45-17.10 | Distances to and the sparsity of lattice points in rational polyhedra<br>Aled Williams | | Symmetric projection attractor reconstruction: Multi-dimensional embedding of physiological signals<br>Jane Lyle |

| Wednesday 15th July 2020 | | | |
|---|---|---|---|
| 10.00-10.50 | The normal Hausdorff spectrum of pro-p groups<br>Anitha Thillaisundaram (Plenary Speaker) | | Optimization Challenges in the Commercial Aviation Sector<br>Jaroslav Fowkes (Plenary Speaker) |
| 10.55-11.30 | Networking shuffle | | |
| 11.35-12.00 | An algebraic Brascamp-Lieb inequality<br>Jennifer Duncan | Commutativity with permiting n-derivations of semiprime rings<br>Mehsin Jabel Atteya | From device to code: Optical physically unclonable functions<br>Elliott Ball |
| 12.10-12.35 | Sparse T1 theorems<br>Gianmarco Brocchi | Drinfeld twists and rational Cherednik algebras<br>Edward Jones-Healey | Coupled nonlinear Schrodinger equations and their applications to nonlinear mechanical topological insulators<br>David Snee |
| Lunch break | | | |
| 14.00-14.50 | Problems session | | |
| 15.00-15.45 | News from the MAGIC Taught Course Centre (Panel Session) | | |
| 16.00-16.25 | Monochromatic cycle partitioning<br>Vincent Pfenninger | What is Homomorphic Encryption and why is it so noisy?<br>Erin Hales | Exact and approximate approaches for stochastic receptor-ligand competition dynamics<br>Polly-Anne Jeffrey |
| 16.35-17.00 | Almost all optimally coloured complete graphs contain a rainbow Hamilton path<br>Stephen Gould | The foundations of infinite-dimensional spectral computations<br>Matthew Colbrook | Population model approximations of dynamics on networks<br>Alice Tapper |
| 17.10-17.35 | | | Creasing of an incompressible, isotropic, hyperelastic material in uni-axial compression: an analytical study<br>Dominic Emery |
| 17.45-18.00 | Closing session | | |

# Networking Sessions

## Networking Session 1 - Group networking session

14th July 10:55-11:30 (35 minutes)

Choose your group by your area of research and join fellow early career researchers within the same field for a great opportunity to network. Discuss common issues, share ideas, or just chat about anything for a chance to get to know one another. In each group, it is recommended that everyone introduces themselves and mentions what their research project is on.

## Networking Session 2 - Beginning a Career in Academia (Panel Session)

14th July 14:55-15:30 (35 minutes)

Hear from our keynote speakers for a fantastic opportunity to learn about their experiences in beginning their careers in academia. We shall discuss making applications for post-doctorate and other positions, applications for funding such positions and what their experiences have been from holding these jobs. This will include a Q&A session.

## Networking Session 3 - Networking Shuffle

15th July 10:55-11:30 (35 minutes)

Join us for this networking shuffle for a great opportunity to meet and discuss with researchers from a variety of mathematical disciplines. Everyone will be given 3 random numbers between 1 and 6 which allocates them to three different groups. Each group discussion will last 10 minutes with an additional 1 minute window to allow everyone to swap between their groups.

## Networking Session 4 - Problems Session

15th July 14:00-14:50 (50 minutes)

In this final networking session, you will be able to present interesting questions and open problems from your research or from outside your current area. The problem session will be split into two parallel zoom meetings allowing separate discussion for applied and pure related problems. Each person will have 2-3 minutes to describe a problem, and viewers will be able to ask questions/suggest ideas. We recommend that presenters provide their email address so anyone interested in the problem can contact you later. For participants who do not wish to present a problem, the session will be an excellent opportunity to find out about open problems in your own research area, and we hope it will encourage collaboration between different institutions and disciplines.

# News from the MAGIC Taught Course Centre (Panel Session)

15th July 15:00-15:45 (45 minutes)

This is an opportunity to meet some of those who are currently managing the MAGIC taught course centre. This session will be hosted by Pete Ashwin, Director of MAGIC (University of Exeter), Yuri Bazlov, Deputy Director of MAGIC (University of Manchester), and Tim Phillips, Chair of MAGIC Programme Committee (Cardiff University). They will outline their plans for the next year, which includes, in particular, moving from Visimeet to Zoom. This will include time for Q&A and would be an excellent opportunity to provide feedback about MAGIC, the courses offered and the plans for the future.

We particularly recommend that participants from universities in the MAGIC group attend this session.

# Plenary Talks

## Pure Mathematics

### On convergence of Fourier integrals

Jonathan Hickman, University of Edinburgh
14th July 10:00-10:50

In the first half of the 20th century great advances were made in understanding convergence of Fourier series and integrals in one dimension. Many natural convergence problems in higher dimensions are still poorly understood, however, despite great attention by many prominent mathematicians over the last four decades. In this talk I will introduce the basic questions, describe their rich underlying geometry, and explain some recent developments in various joint works which have applied tools from incidence and algebraic geometry to these problems.

### Ryser's conjecture and more

Liana Yepremyan, The London School of Economics and Political Science
14th July 14:00-14:50

A Latin square of order $n$ is an $n \times n$ array filled with $n$ symbols such that each symbol appears only once in every row or column and a transversal is a collection of cells which do not share the same row, column or symbol. The study of Latin squares goes back more than 200 years to the work of Euler. One of the most famous open problems in this area is a conjecture of Ryser, Brualdi and Stein from 60s which says that every Latin square of order $n \times n$ contains a transversal of order $n-1$. A closely related problem is 40 year old conjecture of Brouwer that every Steiner triple system of order $n$ contains a matching of size $(n-4)/3$. The third problem we'd like to mention asks how many distinct symbols in Latin arrays suffice to guarantee a full transversal? In this talk we discuss a novel approach to attack these problems. Joint work with Peter Keevash, Alexey Pokrovskiy and Benny Sudakov.

# The normal Hausdorff spectrum of pro-$p$ groups

Anitha Thillaisundaram, University of Lincoln
15th July 10:00-10:50

Hausdorff dimension is a generalisation of integer dimension to non-integer dimension, that was initially applied to fractals in Euclidean space. However, in the 1990s, Hausdorff dimension was applied to certain infinite groups, namely profinite groups. In this talk, we focus on pro-$p$ groups, which are informally speaking, infinite groups that are made up of finite $p$-groups. The normal Hausdorff spectrum of a pro-$p$ group $G$ is the set of Hausdorff dimensions of all closed normal subgroups of $G$. For finitely generated pro-$p$ groups, previously known normal Hausdorff spectra were finite, often just the trivial set $\{0, 1\}$. Shalev asked in 2000 whether there exists a finitely generated pro-$p$ group with infinite normal Hausdorff spectrum, and whether this spectrum can contain an interval. We give a positive answer to both questions, using wreath products, and we state some open problems. This is joint work with Benjamin Klopsch and Iker de las Heras.

# Applied Mathematics

## Microscale to macroscale in suspension mechanics

### Adam Townsend, Imperial College London
14th July 10:00-10:50

Complex fluids appear in many biological and industrial settings. A key feature is that their interesting macroscale behaviour derives from their complex microscale structure. In many cases, these fluids are suspensions - some viscous fluid with particles or fibres suspended within. For example, in polymeric fluids, flexible suspended fibres lead to non-Newtonian bulk responses such as shear thinning or viscoelasticity. When entangled or connected in networks, fibres form gels and disordered solids as is the case in important biological materials such as mucus.

These systems are interesting for mathematicians because the question is raised as to which scale is 'right' to investigate these fluids. The project I have been working on for the last few years - modelling sperm swimming through mucus - argues that large-scale simulation of microscale fibres, interacting with fluid, allows us to bridge this gap.

In this talk, I give a brief overview of suspension mechanics, then I present our progress so far in modelling fibre suspensions - the challenges behind moving to 3D, how we created large networks for sperm-like swimmers to navigate through, as well as the interesting behaviour of fibres in other configurations. I will share my (honest!) experience of using computational infrastructure, as well as presenting avenues for the future.

## MorphoMecanX: mixing (plant) biology with physics, mathematics and computer science

### Gabriella Mosca, University of Zürich
14th July 14:00-14:50

Plants are sessile organisms whose cells, differently from animal ones, are connected to each other through a rigid scaffold (the cell wall), which prevents them from moving around during development. Nonetheless, they develop into a myriad of structures, shapes and sizes starting from very similar configurations. To understand how this is achieved, a holistic approach, where the action of biochemical factors (i.e. genes, hormones) is integrated with mechanics, is required. In fact, mechanical forces are what ensures a compatible configuration in the tissue when growth is non-homogeneous, while mechanical properties and mechanical state act as modulators of the response to biochemically specified growth. MorphoMechanX is an interactive software based on the integration of standard elasticity theory with growth by means of the finite element method (FEM): acting on realistic 3D templates, it allows to explore how growth, as specified by biochemistry, interacts with mechanics allowing to test in silico different developmental scenario.

# Optimization Challenges in the Commercial Aviation Sector

Jaroslav Fowkes, University of Oxford
15th July 10:00-10:50

The commercial aviation sector is a low-margin business with high fixed costs, namely operating the aircraft themselves. It is therefore of great importance for an airline to maximize passenger capacity on its network, a process known as revenue management. Studies have shown that effective revenue management can increase airline revenues by up to $7\%$, which in today's challenging environment can be the difference between making a profit as opposed to a loss. There has been a great deal of research on optimization techniques for revenue management, however the majority of existing work focuses on full-service airlines and their largely outdated capacity allocation models based on customer segmentation. More recently, full-service airlines have started to adopt the dynamic pricing models used by low-cost carriers, where seat price is used to control passenger capacity directly. While dynamic pricing models offer significant benefits over capacity allocation models, they are considerably more challenging to optimize due to the presence of network constraints. In this talk I will review the optimization challenges that arise when solving both capacity allocation and dynamic pricing revenue management models in the context of a large full-service airline.

# Short Talks

## Pure Mathematics

### Strong components of random digraphs from the configuration model: the barely subcritical regime

Matthew Coulson, Universitat Politècnica de Catalunya
14th July 11:35-12:00

One of the foundational questions in the theory of random graphs concerns the size of the largest component of such a graph. For random digraphs an analogous question asks what is the size of the largest strong component? We consider this question in the directed configuration model which is a model for random digraphs with a fixed degree sequence: each vertex $v$ has a specified in-degree $d_v^-$ and out-degree $d_v^+$. Define

$$Q := \frac{\sum_v d_v^- d_v^+}{\sum_v d_v^+} - 1.$$

We show that in the barely subcritical regime, the largest strong component is a directed cycle of size $\Theta(1/|Q|)$ and furthermore, that the probability the $k$th largest strong component has size at most $\alpha/|Q|$ is asymptotically equal to

$$\sum_{i=0}^{k-1} \frac{\xi_\alpha^i}{i!} e^{-\xi_\alpha} \text{ where } \xi_\alpha := \int_\alpha^\infty \frac{e^{-\lambda}}{\lambda} d\lambda.$$

This talk is based on joint work with Guillem Perarnau.

### Large trees in tournaments

Alistair Benford, University of Birmingham
14th July 12:10-12:35

Let $G$ be a tournament - that is, a complete graph with oriented edges - and let $T$ be an oriented tree on $n$ vertices. How large does $G$ need to be in order to guarantee it contains a copy of $T$? Sumner's conjecture says that so long as $G$ has at least $2n-2$ vertices, it will contain a copy of any $n$-vertex tree. A more general conjecture, about which less is known, is that $G$ only needs $n+k-1$ vertices, where $k$ is the number of leaves of the tree. The existing results on these conjectures draw from mixture of techniques including randomised embedding, regularity methods, and absorption techniques. In this talk we will look at some of these results, and discuss progress that could be made, particularly towards the second conjecture.

## Frobenius manifolds in mathematics and beyond

Karoline van Gemst, University of Sheffield
14th July 15:35-16:00

A Frobenius manifold is a complex manifold with a particular algebraic structure on its tangent bundle. Such manifolds were originally introduced by Boris Dubrovin in the early 90's in order to describe the geometry of 2-dimensional topological field theories. Since then, Frobenius manifolds have been found to naturally appear in many different mathematical contexts, for example in singularity theory, quantum cohomology, and mirror symmetry.

In this talk I will introduce the concept of Frobenius manifolds, give some examples, and if time permits, mention my own work on the topic.

## Describing the associated order in a Hopf Galois structure of extensions of $p$-adic fields

Daniel Gil Munoz, Universitat Politècnica de Catalunya
14th July 16:10-16:35

Hopf Galois theory is a generalization of Galois theory in which, for an extension $L/K$, the Galois group with the classical Galois action is replaced by a pair formed by a $K$-Hopf algebra $H$ and a $K$-linear action of $H$ over $L$ such that the canonical map $L \otimes_K H \longrightarrow \mathrm{End}_K(L)$ is an isomorphism. We say that this pair is a Hopf Galois structure or that $L/K$ is $H$-Galois. This approach may be used to broaden the domain of Galois module theory, giving rise to a Hopf Galois module theory. If $L/K$ is an $H$-Galois extension of $p$-adic fields, the associated order $\mathfrak{A}_H$ of $\mathcal{O}_L$ is defined as the maximal $\mathcal{O}_K$-order in $H$ that acts over $\mathcal{O}_L$. In this talk, we present a general method to compute a basis of $\mathfrak{A}_H$ and we focus on the case that $H$ is induced, i.e, it can be expressed in terms of Hopf Galois structures of more simple extensions. This is a joint work with Anna Rio.

## Distances to and the sparsity of lattice points in rational polyhedra

Aled Williams, University of Cardiff
14th July 16:45 - 17:10

During this talk, we show a surprising relation that holds between two well-established areas of research, namely proximity and sparsity of solutions to integer programs. The proximity-type results provide estimates for the distance between optimal vertex solutions of linear programming relaxations and feasible integer points. The sparsity-type results, in their turn, provide bounds on the size of support of feasible integer points and solutions to integer programs.

During this talk, we focus mainly on the *knapsack scenario*. In this context, our results can be viewed as a transference result which allows strengthening the best known distance bound if integer points in the knapsack polytope are not sparse and, vice versa, strengthening the best known sparsity bound if feasible integer points are sufficiently far from a vertex of the knapsack polytope. It should be noted that we have generalised the transference result to general integer programs and this more general result will be briefly outlined for completeness. This is joint work with Iskander Aliev, Marcel Celaya and Martin Henk.

## An algebraic Brascamp-Lieb inequality

Jennifer Duncan, University of Birmingham
15th July 11:35-12:00

The Brascamp-Lieb inequalities are a natural generalisation of many familiar multilinear inequalities that arise in mathematical analysis, classical examples of which include Hölder's inequality, Young's convolution inequality, and the Loomis-Whitney inequality. Given a collection of linear surjections $L_j : \mathbb{R}^n \to \mathbb{R}^{n_j}$ and exponents $p_j \in (0,1)$, we define the associated Brascamp-Lieb inequality as follows:

$$\int_{\mathbb{R}^n} \prod_{j=1}^{m} f_j \circ L_j(x)^{p_j} dx \lesssim \prod_{j=1}^{m} \left( \int_{\mathbb{R}^{n_j}} f_j \right)^{p_j}.$$

It is common in applications to encounter nonlinear variants, where the linear maps $L_j$ are replaced with nonlinear maps between manifolds. By incorporating a weight factor that compensates for local degeneracies, we establish a global nonlinear Brascamp-Lieb inequality for maps that exhibit a certain algebraic structure, with a constant that explicitly depends only on the associated 'degrees' of these maps.

## Sparse T1 theorems

Gianmarco Brocchi, University of Birmingham
15th July 12:10-12:35

In the last decade a plethora of sharp weighted $L^p$-estimates has been obtained for several different operators. These estimates (sharp in the dependence on the characteristic of the weight) follow from a sparse domination of the operator. Roughly speaking, a sparse domination consists in bounding the operator by a positive dyadic form.

It has been shown that Calderón–Zygmund operators and square functions admit such a domination even under minimal $T1$ hypothesis.

In this talk we introduce the concept of sparse domination and we present one of the basic object used in dyadic harmonic analysis: shifted dyadic grids.

## Monochromatic cycle partitioning

Vincent Pfenninger, University of Birmingham
15th July 16:00-16:25

Lehel conjectured that every red-blue edge-colouring of the complete graph admits a vertex partition into a red cycle and a blue cycle. This conjecture was proved by Bessy and Thomassé in 2010.

We consider a generalisation of Lehel's conjecture to hypergraphs. In particular, we prove that every red-blue edge-colouring of the complete 4-uniform hypergraph contains a red and a blue tight cycle that are disjoint and together cover almost all vertices.

This is joint work with Allan Lo.

## Almost all optimally coloured complete graphs contain a rainbow Hamilton path

Stephen Gould, University of Birmingham
15th July 16:35-17:00

A subgraph $H$ of an edge-coloured graph is called rainbow if all of the edges of $H$ have different colours. In 1989, Andersen conjectured that every proper edge-colouring of $K_n$ admits a rainbow path of length $n-2$. We show that almost all optimal edge-colourings of $K_n$ admit both (i) a rainbow Hamilton path and (ii) a rainbow cycle using all of the colours. This result demonstrates that Andersen's Conjecture holds for almost all optimal edge-colourings of $K_n$ and answers a recent question of Ferber, Jain, and Sudakov. Our result also has applications to the existence of transversals in random symmetric Latin squares. This is joint work with Tom Kelly, Daniela Kühn, and Deryk Osthus.

## Blocks of finite groups of tame type

Norman MacGregor, University of Birmingham
14th July 11:35-12:00

We say that an algebra has representation type one of finite, tame or wild, which may loosely be described as easy, medium and hard. A block - an indecomposable summand of the group algebra kG - of a finite group is tame if and only if its defect groups are dihedral, semidihedral or quaternion. Karin Erdmann classified possible tame algebras up to Morita equivalence, and using the classification of finite simple groups we attempt to show precisely which classes occur as blocks of finite groups, primarily using techniques in the representation theory of finite groups of lie type.

## Donovan's conjecture and the classification of block

Cesare Giulio Ardito, University of Manchester
14th July 12:10-12:35

Donovan's conjecture predicts that given a $p$-group $D$ there are only finitely many Morita equivalence classes of blocks of group algebras with defect group $D$. While the conjecture is still open for a generic $p$-group $D$, it has been proven in $2014$ by Eaton, Kessar, Külshammer and Sambale when $D$ is an elementary abelian $2$-group, and in 2018 by Eaton and Livesey when $D$ is any abelian $2$-group. The proof, however, does not describe these equivalence classes explicitly.

A classification up to Morita equivalence over a complete discrete valuation ring $\mathcal{O}$ has been achieved for $D$ with rank $3$ or less, and for $D = (C_2)^4$. I have shown $(C_2)^5$, and I have partial results on $(C_2)^6$. In this talk I will introduce the topic, give the relevant definitions and then describe the process of classifying this blocks, with a particular focus on the methodology and the individual tools needed to achieve a complete classification.

## N-centred Lipschitz Quotient Mappings

Ricky Hutchins, University of Birmingham
14th July 15:35-16:00

We define the notion of a $N$-fold Lipschitz Quotient Mapping (LQM) being $N$-centred and use this property to determine a new upper bound for the ratio of the co-Lipschitz and Lipschitz constants, $c/L$, of such LQM's equipped with polygonal norms on $4m$ sides, $m \in \mathbb{N}$. Further we show that every 2-fold LQM can be identified with a 2-Centred LQM, improving the current known best estimate for the ratio of constants to

$$\frac{c}{L} \leq \frac{1}{2 + (1/65)}.$$

## Limit points of Mahler measures coming from digraphs

Joshua Coyston, Royal Holloway University of London
14th July 16:10-16:35

The Mahler measure of a single variable polynomial $f$ is the geometric mean of $|f(z)|$ on the unit circle. The search to find the smallest Mahler measure, or a complete set of small Mahler measures, has been ongoing for almost 100 years. I will introduce a recent concept - the Mahler measure of a directed graph (digraph) - and how some tools from graph theory have helped us find limit points of Mahler measures.

## Commutativity with permiting $n$-derivations of semiprime rings

Mehsin Jabel Atteya, University of Leicester
15th July 11:35-12:00

The main purpose of this paper is to construct the formula of the composition of $n$-derivations of any ring and investigate some properties of permuting $n$-derivations for prime rings and semiprime rings. This talk is divided into three sections, where, in the first, we elaborate on the formula of the composition of $n$-derivations and in the second we study commutativity with centralizer permuting $n$-derivations for prime rings and semiprime rings. Actually, we examine the action of some identities on prime rings and semiprime rings while the final section focuses on the effect of the trace of permuting $n$-derivations satisfying certain identities.

## Drinfeld twists and rational Cherednik algebras

Edward Jones-Healey, University of Manchester
15th July 12:10-12:35

Rational Cherednik algebras arose as degenerations of Cherednik's double affine Hecke algebras, with both types of algebras finding numerous applications to the theory of integrable systems, combinatorics and algebraic geometry. Recently, rational Cherednik algebras were found to be special cases of a broader class of braided Cherednik algebras. We motivate this generalisation by showing how the groups from which these algebras are built up from satisfy a more general problem coming from invariant theory. In the case of rational Cherednik algebras, these groups are the complex reflection groups. Next I introduce Hopf algebras, and the Drinfeld twist, which provides a way to deform certain algebras into new ones. We finish with a brief discussion of a very recent result that a certain braided Cherednik algebra arises as the Drinfeld twist of the rational Cherednik algebra.

## What is Homomorphic Encryption and why is it so noisy?

Erin Hales, Royal Holloway University of London
15th July 16:00-16:25

Homomorphic Encryption (HE) enables encrypted data to remain private while it is being processed and manipulated. This allows outsourcing of computation and invites possible applications such as cloud computation and privacy-preserving machine learning. Ciphertexts in all HE schemes have some inherent 'noise' that we require for security. However, this noise accrues with each successive operation until the ciphertext becomes too corrupted to decrypt correctly. It is crucial that we develop a good understanding of noise growth if we aim to retain correctness whilst improving the efficiency of HE schemes. In this talk, we will explore the hard problems that underpin HE and how they cause noise. We then consider two ways to manage noise: bootstrapping, which enables us to refresh the noise periodically; and levelled

HE, in which we scale parameters according to noise growth for a particular function. We will discuss the different situations in which to use these methods.

## The foundations of infinite-dimensional spectral computations

Matthew Colbrook, University of Cambridge
15th July 16:35-17:00

Spectral computations in infinite dimensions are ubiquitous in the sciences and computing spectra is one of the most studied areas of computational mathematics over the last half-century. However, such computations are infamously difficult, since standard approaches do not, in general, produce correct solutions. This talk will introduce algorithms that compute spectral properties of operators on separable Hilbert spaces, with a focus on computing spectra and computing spectral measures. The first is solved by an algorithm that computes, with error control, spectra of arguably any operator of applicable interest. The second is solved by computing convolutions of rational kernels with the measure. As well as solving long-standing computational problems for the first time, these algorithms are proven to be optimal, with ramifications beyond spectral theory. Numerical examples will be given, demonstrating efficiency, and tackling difficult problems in fields such as chemistry and physics.

# Applied Mathematics

## The evolution of a three dimensional microbubble in non-Newtonian fluid

Eoin O'Brien, University of Birmingham
14th July 11:35-12:00

Microbubble dynamics in non-newtonian fluids has direct applications in drug delivery, ultrasonic cleaning and weaponry. The jet formation at the collapse of the bubble can lead to high levels of pressure resulting in damage to the surrounding surfaces. Understanding the evolution of these microbubbles as a result of differing geometries or high intensity ultrasonic waves is therefore of the upmost importance.

The evolution of a three dimensional bubble is modelled using potential flow theory, and solved numerically using the boundary integral method. This talk combines both the 3D bubble model used to simulate a microbubble near a rigid boundary subject to an acoustic wave (Q. X. Wang and K. Manmi), with the axisymmetric non-newtonian model (S.J. Lind and T.N. Phillips) to encorporate the effects of a non-newtonian fluid on a three dimensional bubble, subject to an acoustic wave.

## Order from disorder: chaos, turbulence and recurrent flow

Edward Redfern, Keele University
14th July 12:10-12:35

Turbulent fluid phenomena are ubiquitous in nature; from complex weather systems to pipe flow. Understanding the behaviour of these events has captured the minds of scientists for hundreds of years. We search for a way to reduce the complexity of a turbulent flow by building a skeleton of simple unstable solutions which mimic the characteristics of the full flow. To find the bones of this skeleton we search for periodic behaviour in the system by comparing the flow at different times using a numerical simulation. We then converge these near repeats using a type of Newton-Raphson method. This method requires large computational power and does not always converge to a periodic flow. In order to flnd a way to improve this, we consider using the angle between repeating trajectories; close and parallel trajectories are necessary for periodic behaviour. We verify the method with a known periodic flow.

## Layered Bayesian Learning given Highly Discontinuous Data - Application to Materials Science

Georgios Stagakis, Loughborough University
14th July 15:35-16:00

I will discuss the Bayesian learning of the material density function of a sample of material, at any sub-surface location in this sample. This learning is challenged fundamentally by two factors. The first factor is associated with the absence of a training dataset comprising pairs of values of design location and density variables. The second factor is associated with the highly discontinuous nature of the material density function. This training data is generated Bayesianly, via multiple sequential inversions of image data obtained by imaging the material sample with Scanning Electron Microscopes. Once generated, the learning of the material density function is performed by modelling it with a Gaussian Process, using parametrisation of the relevant covariance matrix with non-stationary kernels that are free-form. Empirical illustration on realistically noisy, simulated image data will be discussed, while application on real-world data is underway.

## Analysis of temperature and humidity in Oman using modern statistical method

Safia Al Marhoobi, University of Cardiff
14th July 16:10-16:35

Singular Spectrum Analysis (SSA) is a non parametric, powerful method for time series decomposition and identification. Its uses include, but are not limited to, parametric estimation, forecasting and gap filling. The aim of this work is to demonstrate the use of Basic SSA for extracting annual/daily oscillations for hourly temperature and humidity data. The data used in this study was collected from six stations in Sultanate of Oman from the period of 2009 to 2018. Unfortunately, it contained numerous missing values and, as a result, required the consideration of a number of different gaps filling techniques; these include the iterative approach, multiple regression and regression with lagging. Imputation methods provided a solution with satisfying results and correct the missing data problems. The Mann Kendall test was applied to identify for the presence of monotonous in annual oscillation. Keywords— time series, Singular Spectrum Analysis, missing data, gaps filling, Mann Kendall

## Symmetric projection attractor reconstruction: Multidimensional embedding of physiological signals

Jane Lyle, University of Surrey
14th July 16:45-17:10

Physiological signals are typically analysed by identifying local extrema, but this discards the detail of the waveform morphology. Delay coordinate embedding provides that we can place $N$ points on a waveform and use these to reconstruct an attractor in $N$-dimensional phase space. Our Symmetric Projection Attractor Reconstruction (SPAR) technique adapts this to extract information from the whole waveform of any approximately periodic signal by 3-point embedding and subsequently projects this to a plane to give an image that is easily quantified.

We extend SPAR for $N > 3$ embedding dimensions with a simple generalised result that aids our exploration of the attractor properties, and its relationship to the underlying signal. We exemplify our method by discriminating gender from the electrocardiogram (ECG) using machine learning and quantify an individual's response to cardioactive drugs, supporting a stratified approach to patient management.

## From device to code: Optical physically unclonable functions

Elliott Ball, Lancaster University
15th July 11:35-12:00

Classical security solutions generally rely upon assumptions of certain mathematical tasks being 'hard' to perform, but 'easy' to verify solutions for. This lack of provable security has lead to research of protocols which are provably secure mathematically, using the laws of physics as opposed to unproven assumptions.

One set of manifestations of such research for device authentication is Physically Unclonable Functions (PUFs). Quantum-Optical PUFs are physical devices whose microscopic features and imperfections are inherently random, unique, and unclonable, and these features may be exhibited by their wavelength light emissions. I shall discuss the mathematical challenges associated with capturing the uniqueness inherent to these devices, and the processing of such information for assessment against a suitable set of figures of merit, in order to generate unique fingerprints for device authentication.

## Coupled nonlinear Schrodinger equations and their applications to nonlinear mechanical topological insulators

David Snee, Northumbria University
15th July 12:10-12:35

We show theoretically that the classical 1D nonlinear Schrodinger (NLS) and coupled nonlinear Schrodinger (CNLS) equations govern the envelope(s) of nonlinear edge waves in a 2D mechanical topological insulator (TI). TIs are materials where excitations propagate only on the edge/surface, thus they may have significant implications on the design of acoustic devices.

Theoretical predictions are confirmed by numerical simulations of the original 2D system for various types of edge solitons propagating unidirectionally with an immunity to backscattering at topological defects. Solitons are nonlinear waves that maintain their shape and velocity by balancing nonlinearity and dispersion. These nonlinear structures transmit energy and interact with no physical change in their profile, therefore they have potential applications to collision-based computing. The bimodal CNLS system is non-integrable by nature and as such, we consider new solutions including travelling fronts and vector solitons.

## Exact and approximate approaches for stochastic receptorligand competition dynamics

Polly-Anne Jeffrey, University of Leeds
15th July 16:00-16:25

On the surface (membrane) of human cells there exist many different molecules, namely receptors which can bind with other small molecules called ligands, diffusing in the extracellular medium. These interactions ultimately determine the fate of a cell through intracellular processes which differ depending on the receptor and ligand combination. Often, different receptors can bind a common ligand, where the outcome of this interaction differs according to the receptor type. Hence, there is an element of natural competition for a common ligand between receptors of different types and the dynamics can be modelled mathematically as a competition process. Here I will introduce a stochastic competition process which is appropriate for modelling scenarios in which there are low copy numbers of molecules. I will present two methods of analysis of stochastic descriptors for such a process, one a matrix analytic approach and the other a “moderate competition” approximation.

## Population model approximations of dynamics on networks

Alice Tapper, University of Leeds
15th July 16:35-17:00

The study of spreading processes on networks has relevance across many fields, including biology, social sciences and transport. Real-life applications often involve large, complex networks but the master equation, which governs such spreading dynamics, quickly becomes intractable on networks with even a modest number of nodes. This has motivated the search for approximations that allow the system to be solved efficiently while maintaining accuracy. In this talk I will discuss lumping, a method used to decrease the dimensionality of the system's state space. I will focus on the SIS epidemic model, and show how certain mean-field approximations can be derived as a result of finding the least-squares minimisation of the population model approximate lumping of the system. This result can also be extended to non-binary state dynamics.

## Creasing of an incompressible, isotropic, hyperelastic material in uni-axial compression: an analytical study

Dominic Emery, Keele University
15th July 17:10-17:35

Until very recently, the theoretical nature of elastic creasing as bifurcation phenomenon was shrouded in mystery. This was predominantly due to its widespread misassociation with structural instabilities such as wrinkling and buckling. Such interpretations are inherently flawed; where wrinkling is a linear instability which is physically global, creasing is a non-linear material instability characterised by a geometric localisation of surface self-contact. Indeed, creasing is a global bifurcation phenomenon which gives rise to a *strong* loss of stability. Here we present an overview and critique of recent analytical works predicting this strong stability threshold. We show that the bifurcation triggers the coexistence of an "outer" homogeneously compressed state and an "inner" *infinitesimal* creased region, both of which are matched asymptotically in an intermedial field of incremental deformation. New-found results for the creasing bifurcation threshold are elucidated.