Birmingham Young Mathematician Colloquium

Plenary talks

The Kakeya problem

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The Kakeya problem is an old and important problem in geometric measure theory. It involves computing or estimating the dimension of 'Kakeya sets', which are subsets of n dimensional Euclidean space containing a unit line segment in every direction. A simple example of a Kakeya set is the unit ball, which has positive volume, but Kakeya sets can be much smaller, in particular Kakeya sets exist with zero volume! I will discuss the Kakeya problem, some of the best known results so far, and then give a simple argument yielding a non-optimal (but non-trivial) estimate, which came from some joint work with James Robinson (Warwick) and Eric Olson (Nevada). The talk will be aimed broadly at an analysis audience and I will not assume any prior knowledge of the Kakeya problem or dimension theory.

Analytic approach to large graphs

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The recently emerged theory of combinatorial limits has opened new links between analysis, combinatorics, computer science, group theory and probability theory. Combinatorial limits give an analytic way of representing large discrete objects. In this talk, we will focus on limits of (dense) graphs, and their applications in extremal combinatorics and theoretical computer science. We will particularly discuss graph limits uniquely determined by finitely many density constraints, so-called finitely forcible graph limits. The study of such graph limits, which was greatly advanced by Lovasz and Szegedy, is motivated by problems from extremal combinatorics, and such graph limits are also related to quasirandomness of graphs as studied in the work of Rodl, Thomason, Chung, Graham and Wilson, and others.

Finite p-groups with soft subgroups

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A soft subgroup of a finite group of prime power order is a maximal abelian subgroup which has prime index in its normaliser (Héthelyi). Although it may seem that such condition stems out of the blue, much can be said on the subgroup structure of a finite group of prime power order having a soft subgroup. In this talk, we shall present examples, results and applications about such groups.

Non-linear compressed sensing, from theory to application

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Compressed sensing is a technique used in the acquisition of signals and images that allows signal structure to be exploited when measuring data. This can lead to a significant reduction in the required number of measurements that has to be taken to uniquely characterise the data. Furthermore, compressed sensing theory provides efficient computational strategies that can be used to recover the signals from the measured data.

In this talk I will discuss two recent advances in the field. On the one hand, I discuss theoretical and empirical results that extend the compressed sensing formulation beyond the traditional linear observation model to count for more general non-linear problems. On the other hand, I will discuss practical approaches that combine several constraint sets to better model prior knowledge on data structure, which can significantly improve the performance of compressed sensing methods. Examples will be provided from the field of computed tomography, where a non-linear x-ray model is used to estimate three dimensional x-ray attenuation profiles from two dimensional x-ray projections.

From ants to embryos: applying mathematical modelling to biological systems

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How do ant colonies make decisions? Despite the name, the queen ant is no tyrant. Instead, each individual ant makes decisions about their own behaviours, based on the information they can gather themselves. How do these individual decisions lead to a good choice for the whole population? We will discuss how mathematical modelling can be used to explore how the behaviours of individuals can give rise to group behaviours with two applications: decision-making in ant colonies and the collective motion of cells required for embryo formation.

Singular Capillary Microflows: Modelling, Computation and Scaling

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Understanding the formation of liquid drops, their interaction with solid surfaces and coalescence with surrounding drops is the key to optimising a whole host of technological processes, including a number of emerging microfluidic devices such as 3D-printers and lab-on-a-chip devices. Accurate experimental observation of these phenomena is complex due to the small spatio-temporal scales or interest and, consequently, mathematical modelling and computational simulation become key tools with which to probe such flows.

Drop formation, dynamic wetting and coalescence are all so-called 'singular' capillary flows, in which classical modelling approaches lead to infinite values of flow variables and computation becomes increasingly complex. In this talk, I will describe the mathematical models proposed for this class of flows and the techniques which have been used to obtain both approximate and exact computational solutions. Simulations will reveal (a) the dominant physical mechanisms in these flows, (b) the accuracy of scaling laws proposed for them and (c) a number of previous misconceptions in the published literature.

Finally, if time permits, I will discuss how microscopic modelling frameworks, such as molecular dynamics and kinetic theory, could provide insight into these flows which cannot be obtained from a purely macroscopic approach.

Short talks session

Cooperative Investment in multi-period on GARCH with dynamic programming Approach

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In this paper, we present a cooperative investment portfolio allocation problem in multi-period and continuous time frameworks. It is to suggest that agents can invest their joint capital in the typical portfolio, and then split the subsequent outcome according to their risk-reward preferences. It has been proved that with the given strategy all agents can achieve lower risk with the same probable return or greater than the projected profit by taking the same level of risk, compared to an optimal individual investment strategy. We develop a new scenario generation algorithm and stochastic programming SP model. The model is built on Generalized Autoregressive Conditional Heteroskedastic (GARCH) model. In addition, we apply (GARCH) model designed aiming at describing a volatility clustering where in continuous time the volatility has a highly Heteroskedastic. Then, we solved cooperative investment in multi-period by using dynamic programming technique. We improve a dynamic programming algorithm to deliver the optimal time-consistent cooperative trading strategy, and construct the corresponding efficient frontier. The convexity of the efficient frontier for cooperative investment is also proved.

The RKFIT algorithm for nonlinear rational approximation

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For given matrices $\{A, F\} \subset \mathbb{C}^{N\times N}$ and a vector $\mathbf{v} \in \mathbb{C}^N$, we consider the problem of finding a rational function R_m^* of type (m, m) such that

$$\|F\mathbf{v} - R_m(A)\mathbf{v}\|_2^2 \text{ is small.}$$

We propose an iterative algorithm called RKFIT for its solution. At each iteration RKFIT constructs a rational Krylov space and manipulates an associated Arnoldi decomposition to find better approximations to the poles of R_m^*. In the special case when A and F are diagonal matrices, we can compare RKFIT to the popular vector fitting algorithm by Gustavsen and Semlyen (1999).

Computation of first passage times in two-dimensional models of single neurons

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We introduce a general class of two-dimensional neuron models and explain numerical and, in some special cases, analytical methods that allow for the computation of both moments and distributions of firing times in these models. These methods are used to explain the non-trivial behaviour of the mean firing time as a function of the noise amplitude. It is also explained why the observed effect is not directly related to the well-known occurrence of coherence resonance in single neuron models and why it might be interesting biophysically.

Refinements of Brauer's Induction Theorem for Characters of Finite Groups

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Given a finite group G and an irreducible character χ, Brauer's Induction Theorem states that χ can be expressed as an integer combination of irreducible characters induced from elementary subgroups of G. However, there is a possible refinement to be made to this theorem, by showing that any irreducible character can be expressed as an integer combination of irreducible characters induced from subgroups of index divisible by the degree of the character. The proof, by T. Wilde, is known in the p-solvable case and uses the theory of characters of π-separable groups, developed by Isaacs.

This talk will give an introduction to Representation Theory and enough background to understand the statement of Brauer's Induction Theorem. I will discuss simplifications that can be made using Modular Representation Theory and a possible approach in the general case using the Classification of Finite Simple Groups.

Mathematical and numerical model of water waves generated from a wave maker

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Offshore platforms, wind turbines or ships must be designed to resist the load and stress applied by the waves, whose structure is complex due to non-linearity and the dynamic free surface between water and air. The formulation and the simulation of an accurate mathematical and numerical water wave model would enable us to estimate this load and stress on these structures. In order to test against experiments in a wave basin, this model must also include wave makers to generate the waves.

The linear shallow water equations are derived from a variational principle, in a horizontal plane with a piston wave maker on one side. The model is solved numerically with a space-time Finite Element Method, by using a compatible and consistent space-time discrete variational principle, and the numerical results are compared against a semi-analytical exact standing wave solution. The model is then extended to the non-linear shallow water equations, where the wave maker movement is still linear to keep a fixed domain.

Future directions of this work include extending this model to three dimensions, where the challenge is the unknown time dependent boundary condition due to the free surface of the waves, which requires a moving mesh or coordinates transformation. A second set of wave makers, bottom topography and a beach will also be included to the domain in order to fit the experimental wave basin.

Optimal path and cycle decompositions of dense quasirandom graphs

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Motivated by longstanding conjectures regarding decompositions of graphs into paths and cycles, we prove the following optimal decomposition results for random graphs. Let $0 < p < 1$ be constant and let $G \sim G_{n,p}$. Let $odd(G)$ be the number of odd degree vertices in G. Then a.a.s. the following hold:

(i) G can be decomposed into $\lfloor \Delta(G)/2 \rfloor$ cycles and a matching of size $odd(G)/2$.

(ii) G can be decomposed into $\max\{odd(G)/2, \lceil \Delta(G)/2 \rceil\}$ paths.

(iii) G can be decomposed into $\lceil \Delta(G)/2 \rceil$ linear forests.

Each of these bounds is best possible. We actually derive (i)–(iii) from 'quasirandom' versions of our results. In that context, we also determine the edge chromatic number of a given dense quasirandom graph of even order. For all these results, our main tool is a result on Hamilton decompositions of robust expanders by Kühn and Osthus.

Joint work with Daniela Kühn and Deryk Osthus

Finite Graphs Have Unique n-Fold Pseudo-Hyperspace Suspension

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Let X be a metric continuum. Let n be a positive integer, let $C_n(X)$ be the space of all nonempty closed subsets of X with at most n components and let $F_1(X)$ be the space of singletons of X. The *n-fold pseudo-hyperspace suspension* of X is the quotient space $C_n(X)/F_1(X)$ and it will be denoted as $PHS_n(X)$. In this talk we will show that if X is a finite graph and Y is a continuum such that $PHS_n(X)$ is homeomorphic to $PHS_n(Y)$, then X is homeomorphic to Y.

On irreducible characters of Sylow p-subgroups of groups of Lie type

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Let $q = p^e$, for p a prime and e a positive integer. We denote by G a group of Lie type defined over $\mathbb{F}_q$, and by $U = U(G)$ a Sylow p-subgroup of G.

We are interested in computing the *generic character table* of U, that is to compute the entries of the character table of U using a function whose only argument is q. In particular, we get the number $k(U)$ of irreducible characters of U as a polynomial in q. The problem of parametrizing characters and conjugacy classes is in general very difficult. While the latter has been investigated by S. Goodwin and G. Röhrle, we will just focus on the determination of irreducible characters.

I will first briefly explain some recent history on the problem. Following the work of of F. Himstedt, T. Le and K. Magaard, we will see how the assumption of p being a *bad prime* for U determines a remarkable difference in the parametrization of characters in U. For example, if p is a good prime for U, then the character degrees are always powers of q, while this is not in general true when p is a bad prime.

I will then show my results about the classification of all irreducible characters of $U(G)$ when G is a group of type F_4 and $p \neq 2$. In particular, we exhibit a family of characters of U of degree $q^4/3$ for the bad prime $p = 3$, and we see that the expression of $k(U)$ as a polynomial in q is different if $p \geq 5$ and $p = 3$ respectively.

Steady-state multiplicity in a Solid Oxide Fuel Cell: Constant Ohmic Load Operation

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Solid Oxide Fuel Cells are electrochemical devices which converts Hydrogen (via electrochemical reactions) into electricity and/or heat. They are high temperature fuel cells, and are mainly used for stationary applications on a residential or industrial scale, or in mobile form in the form of auxiliary power units (APU) in vehicles.

This presentation looks at the existence of multiple steady-states in a Solid Oxide Fuel Cell for a constant ohmic load operation. The existence and location of these steady-states are studied for varying operating conditions including changes in gas inlet temperatures, inlet velocities, inlet molar rates, and changes in the heat transfer coefficient (which is assumed constant). Also, it will be shown how electrolyte conductivity plays a huge role into the existence of multiple steady-states within a Solid Oxide Fuel Cell.

The model considered is a lumped parameter model, and takes account of heat and mass transfer in the gas channels, the electrochemical processes within the fuel cell (cell voltage and overpotentials), diffusion and mass transfer within the electrodes, and heat from the solid cell component. Fuel and air velocities are assumed constant.

On Hereditarily Just infinite profinite groups

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A profinite group G is said just infinite if every non-trivial continuous quotient of G is finite. Just infinite groups have been studied widely in the past as they yield interesting counterexamples to many problems in group theory (i.e. Burnside and growth problems). By a theorem of J.Wilson, a non-virtually abelian just infinite profinite group G is either branch (special subgroup of the automorphism group of a rooted tree) or hereditarily just infinite (every open subgroup of G is just infinite). While a lot of work on branch groups has been done, the family of hereditarily just infinite groups still remains very mysterious. In this talk I will give a short overview on just infinite profinite groups and then I'll present some of my results on hereditarily just infinite profinite groups.