Lemma (Szemerédi’s Regularity Lemma: Degree Form)

∀ ε > 0 and ℓ₀ ∈ ℕ there exists L₀ = L₀(ε, ℓ₀) s.t. for every d ∈ [0, 1] and for every graph G on n ≥ L₀ vertices there exists a partition V₀, V₁, . . . , Vℓ of V(G) and a spanning subgraph G' of G, s.t. the following conditions hold:

(i) ℓ₀ ≤ ℓ ≤ L₀;
(ii) d_{G'}(x) ≥ d_G(x) − (d + ε)n for every x ∈ V(G);
(iii) the subgraph G'[V_i] is empty for all 1 ≤ i ≤ ℓ;
(iv) |V₀| ≤ εn;
(v) |V₁| = |V₂| = . . . = |Vℓ|;
(vi) for all 1 ≤ i < j ≤ ℓ we have that (Vᵢ, Vⱼ)_{G'} is an ε-regular pair with density > d or 0.

The reduced graph R of G with parameters ε, d is the graph whose vertices are V₁, . . . , Vℓ and in which VᵢVⱼ is an edge precisely when (Vᵢ, Vⱼ)_{G'} is ε-regular with density > d.