The Absorbing Method: Lecture 5

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Lemma (Szemerédi's Regularity Lemma: Degree Form)

$\forall\ \varepsilon > 0$ and $\ell_0 \in \mathbb{N}$ there exists $L_0 = L_0(\varepsilon, \ell_0)$ s.t. for every $d \in [0, 1]$ and for every graph G on $n \geq L_0$ vertices there exists a partition $V_0, V_1, \ldots, V_\ell$ of $V(G)$ and a spanning subgraph G' of G, s.t. the following conditions hold:

(i) $\ell_0 \leq \ell \leq L_0$;

(ii) $d_{G'}(x) \geq d_G(x) - (d + \varepsilon)n$ *for every* $x \in V(G)$;

(iii) *the subgraph* $G'[V_i]$ *is empty for all* $1 \leq i \leq \ell$;

(iv) $|V_0| \leq \varepsilon n$;

(v) $|V_1| = |V_2| = \ldots = |V_\ell|$;

(vi) *for all* $1 \leq i < j \leq \ell$ *we have that* $(V_i, V_j)_{G'}$ *is an* ε*-regular pair with density* $> d$ *or* 0.

The *reduced graph R of G with parameters ε, d* is the graph whose vertices are $V_1, \ldots, V_\ell$ and in which V_iV_j is an edge precisely when $(V_i, V_j)_{G'}$ is ε-regular with density $> d$.