The Absorbing Method: Lecture 4

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A general approach to absorbing for H-factors

Let H be a graph. Given a graph G, a set $S \subseteq V(G)$ is an H-absorbing set for $Q \subseteq V(G)$, if both $G[S]$ and $G[S \cup Q]$ contain H-factors.

Lemma (Lo and Markström, 2015)

Let $h, s \in \mathbb{N}$ and $\xi > 0$. Suppose that H is a graph on h vertices. $\exists\ n_0 \in \mathbb{N}$ s.t. the following holds. Suppose G graph on $n \geq n_0$ vertices so that, for any $x, y \in V(G)$, there are at least ξn^{sh-1} $(sh-1)$-sets $X \subseteq V(G)$ such that both $G[X \cup \{x\}]$ and $G[X \cup \{y\}]$ contain H-factors. Then $V(G)$ contains a set M so that

- $|M| \leq (\xi/2)^h n/4$;
- *M is an H-absorbing set for any $W \subseteq V(G) \setminus M$ such that $|W| \leq (\xi/2)^{2h} n/(32s^2h^3)$ and $|W| \in h\mathbb{N}$.*

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