University of Birmingham
An Autumn Day of Combinatorics

3rd November 2023

Ramsey goodness of loose paths

Simona Boyadzhiyska

The *Ramsey number* of a pair of graphs (G, H), denoted by $R(G, H)$, is the smallest integer n such that, for every red/blue-coloring of the edges of the complete graph K_n, there exists a red copy of G or a blue copy of H. In the 1980s, Burr showed that, if G is large and connected, then $R(G, H)$ is bounded below by $(v(G) - 1)(\chi(H) - 1) + \sigma(H)$, where $\chi(H)$ is the chromatic number of H and $\sigma(H)$ stands for the minimum size of a color class over all proper $\chi(H)$-colorings of H. We say that G is H-*good* if $R(G, H)$ is equal to this general lower bound. This notion was first studied systematically by Burr and Erdős and has received considerable attention from researchers since its introduction. Among other results, it was shown by Burr that, for any graph H, every sufficiently long path is H-good.

These concepts generalize in the natural way to k-graphs, and in this talk we will explore the notion of Ramsey goodness when G is an ℓ-path for some $1 \le \ell \le k - 1$. We will show that, while long loose paths are not always H-good, they are very close to being H-good for every k-graph H. As we will see, this is in stark contrast to the behavior of ℓ-paths for larger ℓ.

This is joint work with Allan Lo.

A resolution of the Kohayakawa Kreuter conjecture for almost all pairs of graphs

Robert Hancock

We study asymmetric Ramsey properties of the random graph $G(n, p)$. For $r \ge 2$ and a graph H, Rödl and Ruciński (1993-5) provided the asymptotic threshold for $G(n, p)$ to have the following property: whenever we r-colour the edges of $G(n, p)$ there exists a monochromatic copy of H as a subgraph. In 1997, Kohayakawa and Kreuter conjectured an asymmetric version of this result, where one replaces H with a set of graphs $H_1, \ldots, H_r$ and we seek the threshold for when every r-colouring contains a monochromatic copy of H_i in colour i for some $i \in \{1, \ldots, r\}$.

The 1-statement of this conjecture was confirmed by Mousset, Nenadov and Samotij in 2020. We extend upon the many partial results for the 0-statement, by resolving it for almost all cases. We reduce the remaining cases to a deterministic colouring problem.

Our methods also extend to the hypergraph setting.

Joint work with Candida Bowtell (University of Warwick) and Joseph Hyde (University of Victoria).

Colourful Hamilton cycles

Katherine Staden

A classical question in graph theory is to find sufficient conditions which guarantee that a graph G contains a Hamilton cycle. A colourful variant of this problem has graphs $G_1, \ldots, G_s$ on the same n-vertex set, where we think of each graph as having a different colour, and we want to find a Hamilton cycle with a given colouring using colours in $1, \ldots, s$. I will present a proof that this is always possible when each G_i has minimum degree at least $(1/2 + o(1))n$, which is approximately best possible.

This is joint work with Candy Bowtell, Patrick Morris and Yani Pehova.

Recent progress on implicit representations of graph classes

John Sylvester

An adjacency labeling scheme for a class of graphs $\mathcal{C}$ defines, for any n-vertex $G \in \mathcal{C}$, an assignment of labels to each vertex in G, so that adjacency in G is determined by a (fixed) function of the two labels of the endpoints. By a counting argument, if there are $|\mathcal{C}_n|$ many n-vertex graphs in $\mathcal{C}$ then any adjacency labeling scheme needs labels with at least $\log|\mathcal{C}_n|/n$ many bits. If such a scheme exists it is called an implicit representation. The existence of a labeling scheme with largest label $f(n)$-bits is equivalent to the existence of a universal graph of size $2^{f(n)}$.

Many classes of graphs (e.g. minor-closed, bounded twin-width, bounded degeneracy) admit an implicit representation and in 1988 it was conjectured that all classes containing at most $2^{\mathcal{O}(n \log n)}$ many n-vertex graphs (factorial classes) have an implicit representation (in this case of size $\mathcal{O}(\log n)$). Hatami & Hatami recently smashed this conjecture by using the probabilistic method to construct a factorial class requiring $n^{1/2-\epsilon}$-bit labels. I will sketch a proof of their amazing result along with some of our own results on questions of implicit representation for small and monotone classes.

Joint work with Édouard Bonnet, Julien Duron, Viktor Zamaraev & Maksim Zhukovskii.

New approaches for the Brown–Erdős–Sós conjecture

Mykhaylo Tyomkyn

The conjecture of Brown, Erdős and Sós from 1973 states that, for any $k \geq 3$, if a 3-uniform hypergraph H with n vertices does not contain a set of $k+3$ vertices spanning at least k edges then it has $o(n^2)$ edges. The case $k = 3$ of this conjecture is the celebrated $(6,3)$-theorem of Ruzsa and Szemerédi, but for $k \geq 4$ the conjecture is wide open.

I will survey some recent progress towards the conjecture: an algebraic relaxation, a Ramsey relaxation, and a new connection to bipartite Turán problems.