# Revision Notes for Perturbation Theory and Asymptotics

These notes are intended to be a rough guide to what you should be revising. They are not by any means complete and as always the material in the exam can come from any method or analysis presented in lectures.

- You should be familiar with the concept of the 'order' symbol and able to find or show that a function $f(x)$ is, for example, $f(x) = O(x^\alpha)$ for some $\alpha$ much like the first problem sheet.
- You should learn the definitions and in particular note that the limits are important when talking about orders.
- You should be able to use a given asymptotic sequence to represent a function by its asymptotic expansion using the given sequence. We did a few examples of these in lecture and this will make good practice.
- You should be confident in deriving asymptotic expressions of functions which have a small parameter $\epsilon$, such as

  $$f(x;\epsilon) = \frac{x\sqrt{1+\epsilon x}}{x+\epsilon}, \qquad x \geq 0,$$

  with $\epsilon \to 0^+$ where it is important to examine situations when $x = O(1)$ and then consider cases in which this expansion breaks down - then you need to rescale etc.
- You should appreciate the main idea behind looking at asymptotic forms of integrals - why do the methods work in general? In particular why is taking the upper limit of some integrals to infinity helpful and why does it not matter that the McLaurin expansion of the integrand may not be valid within the integration range?
- You should carefully consider Watson's Lemma in detail and be familiar with all the main ideas behind the Lemma. Note the conditions that are required for a general integral to have Watson's Lemma remain valid. You should make your way through the proof of Watson's Lemma and be familiar with this - in lectures it has been broken into four parts - this can easily be learnt and will suffice for a proof of Watson's Lemma.
- There are numerous examples given in lectures about Watson's Lemma including its extension - make yourself comfortable with these.
- The same applies to Laplace's method - make yourself familiar with **all** the examples and in particular the longer ones we considered in lectures or in the notes. Do not fall into the trap of reading the notes and just following and nodding your head - make sure you can do the question **all** by yourself - that is without looking at the notes and remember you should be able to follow **all** the points. It is common to be able to follow easily but not 'do' with similar ease - the most helpful thing is to break each question - especially the longer ones - into key parts or points and then **understand** why they work.

- Understand the method of stationary phase - this is slightly different to Laplace's method and works for a different reason. Remember now you are looking for a stationary point and in cases where a stationary point does not exist you may use integration by parts.
- Make yourself familiar with the contour integration we considered in lectures.
- The work we did on boundary layers is important for this course and it is important for you to appreciate the key steps as outlined during lectures. These are; finding the location of the boundary layer, finding the outer solution, rescaling to find the dominant terms, finding the inner solution, matching the two solutions and then forming the composite solution. Knowledge of the finding the dominant balance especially when the coefficients of the differential equation involve arbitrary constants will be very helpful - along the lines of the assessed problem sheet.
- We did a few examples of Linstedt Poincare and this should be examined carefully to learn all the details.
- The method of multiple scales is an important aspect of this course - the method was considered in great detail. The main idea is to determine a uniformly valid solution - usually one term is only sought and you need to ensure that secular terms do not appear. We will go through an example during our revision lectures to supplement examples given in notes and lectures.

I cannot emphasise how important it is to go through **all** problem sheets and assessed worksheets and be able to work through them independently. The first thing you should try to do is ensure that all examples provided have been revised and carefully studied. Thereafter, you should try to learn the finer details of the method themselves. Remember marks will be awarded for method and you should write down anything you think might be helpful to you when you are answering questions - as the marker I am looking for as many pointers to suggest you appreciate or know the answer. Please turn up for the Revision Lecture as this will be useful.
Lastly, I would wish everyone good luck with their exams but if you need luck you're already in trouble :)

JU 03/04/13.