MSM3A05a/MSM4A05a Problem Sheet 4.

QUESTION 1. Show that as $x \to \infty$

$$J_0(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x\cos\theta)d\theta \sim \sqrt{\frac{2}{\pi x}}\cos\left(x - \frac{\pi}{4}\right).$$

QUESTION 2. Consider the integral

$$I(x) = \int_a^b f(t)e^{ixg(t)}dt,$$

where $g(t)$ has a single stationary point at $t = t_0$ where $a < t_0 < b$ and $g''(t_0) = 0$ with $g'''(t_0) \neq 0$. Use the method of stationary phase to show that

$$I(x) \sim \frac{2}{3}f(t_0)\exp\left(ixg(t_0) + i\pi\frac{\Omega}{6}\right)\Gamma\left(\frac{1}{3}\right)\left(\frac{6}{x|g'''(t_0)|}\right)^{\frac{1}{3}},$$

as $x \to \infty$ where $\Omega = sgn(g'''(t_0))$.

QUESTION 3. Find the leading order behaviour of the Bessel function

$$J_n(n) = \frac{1}{\pi}\int_0^{\pi}\cos(n\sin t - nt)dt,$$

as $n \to \infty$.

QUESTION 4 Consider the boundary value problem

$$\epsilon y'' + (1+x)y' + y = 0, \qquad y(0) = 1, y(1) = 1.$$

Use an *outer expansion*

$$y_{out}(x) \sim y_0(x) + \epsilon y_1(x) + \epsilon^2 y_2(x) + \cdots$$

and only the boundary condition $y(1) = 1$ to find y_0, y_1 and y_2.

JU 18/11/12.