MSM3A05b/MSM4A05b Solution Sheet 1. Nonlinear systems and Chaos

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QUESTION 1. In these cases the solution is very similar to the Cantor middle-thirds set. For case (a) the numbers would be represented in base-4 with coefficients 1 and 2 absent and with 0 and 3 present. Similarly, in (b) we have a base-5 representation where none of the terms are 1 or 3.

QUESTION 2. This leads curiously to something that resembles the Serpienski Carpet.

QUESTION 3. The number with ternary representation

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is an element of the middle thirds Cantor set since it consists of repetitions of 0's and 2's and an argument similar to that shown in lectures will suffice to answer the question. To calculate the value of this number we form the infinite geometric progression

$$\frac{2}{3^2} + \frac{2}{3^4} + \frac{2}{3^6} + \cdots = 2\left(\frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} + \cdots\right) = 2\frac{\frac{1}{9}}{1 - \frac{1}{9}} = 2\frac{1}{8} = \frac{1}{4}.$$

The number 3/4 is 3 multiples of the above number and so corresponds to a shift to the left of the above ternary form and so 3/4 in ternary form is given by

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QUESTION 4.

In this case the set is covered by 2^n intervals of length $1/4^n$. Here we may set $\epsilon_n = 1/4^n$ and we have $N(\epsilon_n) = 2^n$ and so

$$D = -\lim_{n\to\infty} \frac{\ln(2^n)}{\ln(4^{-n})} = \frac{1}{2}.$$

QUESTION 5. We require the solution to

$$\left(\frac{1}{2}\right)^d + \left(\frac{1}{4}\right)^d = 1 \to x^2 + x - 1 = 0,$$

where $x = \left(\frac{1}{4}\right)^d$. This has solution $x = 0.5(1 \pm \sqrt{5})$ and hence since we require d to be positive we need

$$d = \frac{\ln(0.5(1 + \sqrt{5}))}{-\ln 4} = 0.3471.$$

JU 17/01/12.