MSM3A05b/MSM4A05b Problem Sheet 5. Nonlinear Systems and Chaos

QUESTION 1.

(a) For the Lorenz equations show that the characteristic equation for the eigenvalues of the Jacobian matrix at the fixed points C^+ and C^- (i.e., these are the fixed points of the Lorenz system not including the origin) is

$$\lambda^3 + (\sigma + b + 1)\lambda^2 + (r + \sigma)b\lambda + 2b\sigma(r - 1) = 0.$$

(b) By seeking solutions of the form $\lambda = i\omega$ where ω is real show that there is a pair of pure imaginary eigenvalues when $r = r_H = \sigma\left(\frac{\sigma+b+3}{\sigma-b-1}\right)$. Why do we need to take $\sigma > b + 1$.
(c) Find the third eigenvalue

QUESTION 2. The Earths magnetic field undergoes constant reversal over long periods of time. In 1958 Rikitake produced a model (referred to as Rikitake's geomagnetic reversal model) to explain the self generation of the Earth's magnetic field by the large current-carrying eddies in the Earth's core. In this respect we consider the equation[1]

$$\begin{aligned} \dot{x} &= -vx + zy, \\ \dot{y} &= -vy + (z - a)x, \\ \dot{z} &= 1 - xy, \end{aligned}$$

where $a, v > 0$ are parameters.
(a) Show that this system is dissipative.
(b) Find the fixed points of this system.
(c) Classify these fixed points.

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[1] This is taken from Strogratz (1994)