MSM3A05b/MSM4A05b Problem Sheet 3. Nonlinear systems and Chaos

QUESTION 1.

Find the homoclinic paths of $\ddot{x} - x + 3x^5 = 0$, $\dot{x} = y$. Show that the time solutions are given by

$$x = \pm\sqrt{(t - t_0)}.$$

You may use the relation

$$\int \frac{1}{\sqrt{u^2 - a^2}} du = \cosh^{-1}\left(\frac{u}{a}\right).$$

QUESTION 2. (a) Write down the system of ordinary differential equations

$$\begin{aligned} \frac{dx}{dt} &= x - y - y^3 - 2x^5 - 2x^3y^2 - xy^4, \\ \frac{dy}{dt} &= x + y + xy^2 - 2yx^4 - 2y^3x^2 - y^5, \end{aligned}$$

in polar coordinates (r, θ) and use the Poicare-Bendixson theorem to show that at least one limit cycle solution exists.

QUESTION 3. Using Bendixson's criterion show that the response amplitudes a, b for the van der Pol equation, that is

$$\dot{a} = \frac{\epsilon}{2}\left(1 - \frac{r^2}{4}\right)a - \frac{\omega^2 - 1}{2\omega}b, \quad \dot{a} = \frac{\epsilon}{2}\left(1 - \frac{r^2}{4}\right)b - \frac{\omega^2 - 1}{2\omega}a + \frac{\Gamma}{2\omega},$$

has no closed paths in the circle $r < \sqrt{2}$.

JU 16/02/11.