MSM3A05b/MSM4A05b Problem Sheet 1. Nonlinear Systems and Chaos

QUESTION 1.

Consider the construction of another type of Cantor set such that
(a) At each stage you remove the middle two quarters of every interval so that $\mathcal{C}_1 = [0, \frac{1}{4}] \cup [\frac{3}{4}, 1]$ or
(b) At each stage remove the second and fourth fifth of every interval so that $\mathcal{C}_1 = [0, \frac{1}{5}] \cup [\frac{2}{5}, \frac{3}{5}] \cup [\frac{4}{5}, 1]$.
Explain how each of these 'Cantor sets' (produced by continuously iterating this procedure) relate to the base-4 and base-5 expansion of numbers?

QUESTION 2. Construct a Pascal triangle as follows:
The top row is 1 and will be labelled as row 0.
The next row is 1 1 and will be labelled as row 1.
Thereafter each row is the sum of the two entries directly above it. If there is no number to the left or the right above the entry place a 1 there. Construct the first 13 rows (you should realise that this is connected to the Binomial expansion of $(1+x)^n$. Now place a square grid box around each number in the Pascal triangle and colour each grid square black if the number is odd and leave blank if it is even. What do you see?

QUESTION 3.

Show that the number with ternary representation

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is an element of the middle thirds Cantor set. Also, find the value of this number. Additionally, write down the number 3/4 in ternary form.

QUESTION 4.

What is the dimension of the Cantor set defined in Q1(a) above?

QUESTION 5. Determine the dimension of the two-scale Cantor set for the case $r_1^2 = r_2 = \frac{1}{4}$.

JU 16/01/13.