MSM3A05b/MSM4A05b Assessed Problem Sheet 2. Nonlinear Systems and Chaos: To be handed in on Monday 11th March at 10am

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QUESTION 1.
[22 MARKS] Consider the self-excited oscillator

$$\frac{d^2x}{dt^2} + \epsilon f(x)\frac{dx}{dt} + x = 0,$$

where

$$f(x) = \begin{cases} -1 & : \quad -1 \leq x \leq 1 \\ 1 & : \quad |x| > 1 \end{cases}$$

(a) Use the method of averaging to find that the limit cycle is given by

$$x(t;\epsilon) = a\cos(t - t_0),$$

as $\epsilon \to 0$.
You need to find an expression for a. Also identify the range of validity of this expression.
(b) Show that the equation is equivalent to the system

$$\frac{dw}{dt} = \frac{x}{\epsilon}, \qquad \frac{dx}{dt} = -\epsilon(w + F(x)),$$

where $F(x) = -x$ for $|x| \leq 1$ and$F(x) = x - 2sgn(x)$ for $|x| > 1$.
(c) Sketch the curve $w = -F(x)$ in the (w, x)-plane and indicate the locus of the limit cycle as $\epsilon \to \infty$.
(d) Find the period $T(\epsilon)$ of the limit cycle as $\epsilon \to \infty$

JU 28/02/13.