MSM3A05a/MSM4A05a Solutions to Assessed Sheet 1. Nonlinear systems and Chaos 2012.

QUESTION 1.
10 MARKS

Let (x_1^*, x_2^*) be a period-2 cycle of the map $f(x)$
(a) Show that $(f^4)'(x_1^*) = (f^4)'(x_2^*) \geq 0$.

Answer: We have that

$$(f^4)'(x) = f'(x)f'(f(x))f'(f^2(x))f'(f^3(x)).$$

Since (x_1^*, x_2^*) is a period-2 cycle of the map $f(x)$ we have $f(x_1^*) = x_2^*$, $f(x_2^*) = x_1^*$, $f^2(x_1^*) = x_1^*$ and $f^2(x_2^*) = x_2^*$, $f^3(x_1^*) = x_2^*$, $f^3(x_2^*) = x_1^*$ etc and so we have

$$(f^4)'(x_1^*) = (f^4)'(x_2^*) = (f'(x_1^*)f'(x_2^*))^2 \geq 0,$$

as required.

(b) Show that (x_1^*, x_2^*) is a stable (unstable) period-2 cycle of $f(x)$ *if and only if* x_1^* and x_2^* are stable (unstable) fixed points of $f^4(x)$.
Answer: The above equation implies that

$$|(f^4)'(x_1^*)| = |(f^4)'(x_2^*)| < 1(> 1) \qquad \text{iff} \qquad |f'(x_1^*)f'(x_2^*)| < 1(> 1),$$

and so since the first expression is a stability test for $f^4(x)$ and the second expression is a stability test for period-2 orbits of $f(x)$ we have shown the above statement.

QUESTION 2.
4 MARKS

Write a paragraph about Felix Hausdorff and his contribution to mathematics. Also mention the differences between the Hausdorff dimension and the box-counting dimension.

Answer: The main difference is that it uses covering sets of varying dimensions and has nicer mathematical properties.

JU 26/01/12.