MSM3A05a/MSM4A05a Assessed Sheet 1. Nonlinear systems and Chaos 2012. Due in 18th Feb at 10am.

QUESTION 1.
10 MARKS

Let (x_1^*, x_2^*) be a period-2 cycle of the map $f(x)$
(a) Show that $(f^4)'(x_1^*) = (f^4)'(x_2^*) \geq 0$.
(b) Show that (x_1^*, x_2^*) is a stable (unstable) period-2 cycle of $f(x)$ *if and only if* x_1^* and x_2^* are stable (unstable) fixed points of $f^4(x)$.

QUESTION 2.
4 MARKS

Write a paragraph about Felix Hausdorff and his contribution to mathematics. Also mention the differences between the Hausdorff dimension and the box-counting dimension.

QUESTION 3.
10 MARKS

Recall that the Tent Map $T_c(x)$ is given by

$$T_c(x) = c\min(x, 1-x) = \begin{cases} cx & x \leq \frac{1}{2}, \\ c(1-x) & x > \frac{1}{2} \end{cases}$$

Now express $T_c^2(x)$ for $0 < c \leq 1$ and in particular show that

$$T_c^2(x) = \begin{cases} c^2 x & 0 \leq x \leq \frac{1}{2c}, \\ c - c^2 x & \frac{1}{2c} \leq x \leq \frac{1}{2}, \\ c - c^2 + c^2 x & \frac{1}{2} \leq x \leq 1 - \frac{1}{2c}, \\ c^2(1-x) & 1 - \frac{1}{2c} \leq x \leq 1, \end{cases}$$

when $1 < c \leq 2$. Hence, or otherwise find all the fixed points of $T_c^2(x)$ for $1 < c \leq 2$ and find all the period-2 orbits $\{x_1^*, x_2^*\}$ and show that $T_c(x_1^*) = x_2^*$ and $T_c(x_2^*) = x_1^*$.

QUESTION 4.
4 MARKS

Show that for the Logistic map $F_\mu(x) = \mu x(1-x)$ the smallest value of μ at which there is a period-3 orbit is $\mu = 1+\sqrt{8}$. [Hint: You can do this with a little thought or you could search the literature out there - i.e. do some research!!!!!]

JU 26/01/13.