Mini-courses

Kaj Nyström
Uppsala Universitet
Parabolic equations with complex coefficients: square function estimates, the Kato square root problem and L^2 solvability of boundary value problems

In this mini-course we will discuss recent developments concerning square function estimates, the Kato square root problem and L^2 solvability of boundary value problems, for parabolic operators of the form

$$\partial_t + \mathcal{L}, \quad \mathcal{L} = -\operatorname{div} A(X,t)\nabla,$$

in $\mathbb{R}^{n+2}_+ := \{(X,t) = (x, x_{n+1}, t) \in \mathbb{R}^n \times \mathbb{R} \times \mathbb{R} : \ x_{n+1} > 0\}$, $n \geq 1$. Concerning A we assume that A is a $(n+1) \times (n+1)$-dimensional matrix which is bounded, measurable, uniformly elliptic and complex.

Javier Parcet
Instituto de Ciencias Matemáticas
Fourier L_p summability with frequencies in nonabelian groups

The L_p-convergence of (smoothly) truncated Fourier integrals is understood for nilpotent Lie groups through the remarkable work of Christ, Müller, Ricci, Stein and others on Hörmander–Mihlin type theorems for this class of groups. It is very natural to analyze what happens when the underlying group hosts the frequency variables, instead of the space ones. Initially considered by Haagerup, Cowling and de Canniere for $p = \infty$, it has become a crucial tool to study approximation properties on the corresponding group von Neumann algebra which ultimately help in their classification. In recent years —motivated from harmonic analysis and operator algebra problems— we have experienced a promising development of the L_p-theory in this direction. I will review these results putting the emphasis on the interaction between harmonic analysis and geometric group theory.

Regional Meeting talks

Pascal Auscher
Université Paris-Sud
Some mathematical legacy from Alan McIntosh

Our dear colleague, Alan McIntosh, passed away a month ago. We will miss his kindness, friendship and spirit. He had a tremendous influence on the fields of analysis, operator theory and partial differential equations. In this talk, I would like to present some of his key contributions and ideas.

Charles Batty
University of Oxford
Holomorphic functions which preserve holomorphic semigroups

Operator semigroups provide an abstract approach to various types of PDEs, particularly diffusion equations, involving a time variable and a generator A which is typically a differential operator in space variables. The greatest regularity of the solutions occurs when the semigroup is holomorphic in the time-variable. The generators of such semigroups are known as sectorial operators. There are many situations where one wishes to replace the generator A by $f(A)$ for some holomorphic function f. For example, Bochner's notion of subordination in probability corresponds exactly to this procedure for the class of Bernstein functions (various other names are used for the same class). Thus it is natural to ask when $f(A)$ is sectorial. This talk will discuss versions of this question and provide some answers.

Tony Carbery
University of Edinburgh
Multilinear duality

We discuss the issue of duality in the context of multilinear rather than linear operators.

Workshop talks

Andrea Carbonaro
Università degli Studi di Genova

Bounded H^∞-calculus for generators of analytic contraction semigroups on L^p spaces

Suppose that $\mathbf{T} = (T(t))_{t>0}$ is a *contraction semigroup* on L^p, $1 \leq p \leq \infty$. Suppose further that $\mathbf{T}$ extends to an analytic contraction semigroup on L^2. In this talk I will discuss the functional calculus problem for the negative generator $\mathscr{A}_p$ of the semigroup $\mathbf{T}$ on L^p, $1 < p < \infty$.
More specifically, I will consider two cases.

1. When $\mathscr{A}_2$ is symmetric, I will show that $\mathscr{A}_p$ has bounded H^∞-calculus in any cone $\{z \in \mathbb{C} \setminus \{0\} : |\arg(z)| < \phi_p^* + \epsilon\}$, $\epsilon > 0$, where $\phi_p^* = \arcsin|1 - 2/p|$ is optimal.

2. In the nonsymmetric case, I can not show you any general result. I will consider instead the particular case when $\mathscr{A} = \mathscr{L}$ is a nonsymmetric finite or infinite dimensional Ornstein-Uhlenbeck operator. If $-\mathscr{L}$ generates an analytic contraction semigroup on $L^2(\gamma_\infty)$, then $\mathscr{L}$ has bounded H^∞-calculus on $L^p(\gamma_\infty)$ in any cone of angle $\theta > \theta_p^*$, where γ_∞ is the associated invariant measure and θ_p^* is the sectoriality angle of $\mathscr{L}$ on $L^p(\gamma_\infty)$. The angle θ_p^* is optimal.

The talk is based on joint works with Oliver Dragičević. The main tool in our proofs is the analysis of the complex time heat flow associated with a particular Bellman function.

Martin Dindoš
University of Edinburgh

Boundary value problems for elliptic PDEs with coefficients satisfying Carleson condition

I will talk on two papers, the first one is an older work with J. Pipher and S. Petermichl and the second one is a joint work with J. Pipher and D. Rule. I will introduce the L^p Dirichlet, Regularity and Neumann problems for elliptic PDEs with bounded measurable coefficients. In general without any additional condition there are examples of elliptic operators such that these BVP are not solvable for any $1 < p < \infty$. I will motivate and introduce a natural extra mild assumption on the coefficients and proceed to present results on solvability of these BVP under this extra conditions.

At the end I will mention few open problems and briefly talk about possible generalizations of these results to elliptic systems.

Véronique Fischer
University of Bath
Multipliers on compact Lie groups

In this talk, I will present conditions on a Fourier multiplier of a compact Lie group which ensure that the corresponding operator is L^p-bounded. They imply the well-known case of spectral multiplier in the Laplace–Beltrami operator, thereby showing that the conditions are sharp. Old and new questions on the subject will be discussed.

Dorothee Frey
Delft University of Technology
Optimal weighted estimates for operators beyond Calderón-Zygmund theory

We shall discuss sharp weighted norm estimates for generalised singular integral operators, such as Riesz transforms and multipliers associated with a second order elliptic operator. The result in particular extends the A_2 conjecture to singular integral operators whose kernel does not satisfy any regularity estimate. The proof is based on a domination by adapted sparse operators. This is joint work with F. Bernicot and S. Petermichl.

José María Martell
Instituto de Ciencias Matemáticas
Free boundary results for real elliptic operators with variable coefficients

The classical theorem of F. and M. Riesz established absolute continuity of harmonic measure with respect to arc length measure, for a simply connected domain in the complex plane with a rectifiable boundary. In this talk we will present some quantitative converse results which are also valid for a class of elliptic operators with variable coefficients. More precisely, let $\Omega \subset \mathbb{R}^{n+1}$, $n \geq 2$, be an open set with Ahlfors–David regular boundary and assume that Ω satisfies the Harnack Chain condition plus an interior (but not exterior) Corkscrew condition (these are quantitative or scale-invariant versions of path-connectedness and openness respectively). We find real symmetric second order divergence form elliptic operators in such a way that if the associated elliptic measure in Ω is absolutely continuous with respect to surface measure on $\partial\Omega$, with scale invariant higher integrability of the Poisson kernel, then Ω has exterior corkscrews. Hence, Ω is indeed an NTA domain and, and since $\partial\Omega$ is Ahlfors–David regular, it follows that $\partial\Omega$ is quantitatively rectifiable.

Joint work with J. Cavero, S. Hofmann, and T. Toro.

Sylvie Monniaux
Aix–Marseille Université

First order approach to L^p estimates for the Stokes operator on Lipschitz domains

In this talk, I will describe a first order approach to developing an L^p theory for the Hodge-Laplacian and the Stokes operator with Hodge boundary conditions, acting on a bounded open subset of $\mathbb{R}^n$. In particular, conditions on the domain and p under which these operators have bounded resolvents, generate analytic semigroups, have bounded Riesz transforms, or have bounded holomorphic functional calculi will be given. The first order approach of initially investigating the Hodge–Dirac operator, provides a framework for strengthening known results and obtaining new ones on general classes of domains, in what we believe is a straightforward manner.

This is a joint work with Alan McIntosh, and I will dedicate this talk to him.

Detlef Müller
Christian-Albrechts-Universität zu Kiel

On sharp constants in local and global Hausdorff–Young inequalities

The best constant for the Hausdorff-Young inequality

$$\|\hat{f}\|_{p'} \leq C_{p,d}\|f\|_p, \quad 1 \leq p \leq 2,$$

on $\mathbb{R}^d$ has been identified by Babenko (when p' is an even integer) and Beckner (for general p) as $C_{p,d} = B_p^d$, with $B_p = (p^{1/p}/p'^{1/p'})^{1/2}$.

Several authors have studied the analogous problem for the group Fourier transform on classes of locally compact groups G, in particular solvable and especially nilpotent Lie groups. For instance, Klein and Russo have established the analogue of Babenko's estimate on the Heisenberg group, with best constant again given by $B_p^{\dim G}$, and partial results for more general values of p on nilpotent groups have been established in work by Russo, by Inoue, and by Baklouti, Ludwig and Smaoui.

On the other hand, for instance for compact groups, it is easily seen that the best constant is 1. However, it seems plausible that, on compact Lie groups G, in the limit as the support of the function f shrinks to the identity element, the best constant will again tend towards $B_p^{\dim G}$ ("sharp local Hausdorff–Young").

In this talk, I shall discuss some new results in this context. This includes the sharp local Hausdorff–Young estimate for central functions f on compact Lie groups, a corresponding local estimate for the Weyl transform for radial functions, and a sketch of the proof that $B_p^{\dim G}$ is a lower bound for the best constant on any (unimodular) Lie group G.

This is joint work in progress with Michael Cowling, Alessio Martini, and Javier Parcet.

Fulvio Ricci
Scuola Normale Superiore
Algebras of singular integral operators on nilpotent groups

We present results obtained jointly with A. Nagel, E.M. Stein and S. Wainger on certain classes of singular integral convolution operators which are closed under composition. These classes are intermediate between the class of Calderón–Zygmund operators with given homogeneity and that of product-type operators. They consist of pseudo-local operators and allow to describe compositions of Calderón–Zygmund operators with different homogeneities and compactly supported kernels.

Maria Vallarino
Politecnico di Torino
Singular integrals on NA groups

Let $G = N \rtimes A$, where N is a stratified Lie group and $A = \mathbb{R}$ acts on N via automorphic dilations. G is a nonunimodular Lie group of exponential growth. Consider a basis of the first layer of the Lie algebra of N and a standard basis of the Lie algebra of A. Lift these vector fields to left invariant vector fields $X_0, \ldots, X_q$, which generate the Lie algebra of G and define a subRiemannian structure on G. We develop a a Calderón-Zygmund theory and introduce a Hardy type space $H^1(G)$ adapted to the subRiemannian metric and the right Haar measure μ of G. These can be used to study boundedness properties of singular integral operators on the group.

In particular, we consider the subLaplacian $\Delta = -\sum_{i=0}^{q} X_i^2$, which is self-adjoint on $L^2(\mu)$. We study the boundedness of spectral multipliers of Δ and Riesz transforms associated with Δ on $L^p(\mu)$, $p \in (1, \infty)$ and on the Hardy space $H^1(G)$.

This is a joint work with Alessio Martini and Alessandro Ottazzi.

Jim Wright
University of Edinburgh
On a problem of Kahane

In the mid to late 1950's Kahane proposed the problem of studying mappings of the circle which transport via composition functions with an absolutely convergent Fourier series to ones with a uniformly convergent Fourier series. We investigate this problem in higher dimensions.