Probabilistic Methods in Graph Theory

University of Birmingham, 25-29 March 2012

Programme

All talks will be in Lecture Room C of the Watson Building (R15 on the map).

	Monday	Tuesday	Wednesday		Thursday
09:00	Dan Král	Alexandr Kostochka	Demetres Christofides	09:00	David Conlon
09:40	Lutz Warnke	Tobias Müller	Kostas Panagiotou	09:40	Jan Hladký
10:20	Yury Person	Andrew Treglown	John Talbot	10:40	Viresh Patel
11:20	Heidi Gebauer	Hiep Hàn	Klas Markström	11:20	Imre Leader
12:00	Anusch Taraz	Wojciech Samotij	Frédéric Havet		
14:30	Mark Walters	Graham Brightwell	Oleg Pikhurko		
15:10	Jan Volec	Dan Hefetz	Peter Allen		
16:10	Mihyun Kang	Béla Csaba	Julia Böttcher		
16:50	Boris Bukh	Jozef Skokan	Stefanie Gerke		

Meal Arrangements

Monday evening:
Dinner will be in Hornton Grange (G13 on the map) at 7pm.

Tuesday evening:
Dinner will be in the Business School/University House (O3 on the map) at 7pm.

Wednesday evening:
Dinner will be in Hornton Grange (G13 on the map) at 7pm.

We have not made any arrangements for lunch; there are several convenient possibilities on and near campus.

- There is a self-service cafeteria on the second floor of the Staff House (R24 on the map); we have reserved some tables here for the use of workshop delegates. The bar ('Bratby Bar') on the ground floor also serves hot food at lunchtime.
- Next door to this, the University Centre (R23 on the map) contains a SPAR and a farm shop selling sandwiches and snacks, as well as 'Café Spice' and 'Café Go' serving hot food in the food court. In the Guild of Students (O1 on the map) there is another SPAR and a Subway.
- Off-campus, there are several shops, pubs and restaurants on Bristol Road (bottom right corner of the campus map, all are located westwards of the South Gate).

Note that the map is divided into red, green, orange and green zones. The first letter of a building indicates the zone where it is located. For example the Conference Park accommodation is in G16 and G14.

List of abstracts

Monday

Daniel Král (Prague): *Domination in 3-edge-colored complete graphs*

Erdős, Faudree, Gould, Gyárfás, Rousseau and Schelp proved that for every complete graph of order n with edges colored with three colors, there exist a set X of 22 vertices and a color c such that the number of vertices in X or joined to a vertex of X by an edge of color c is at least $2n/3$. They also conjectured that the bound of 22 can be lowered to 3. We improve the bound to 4.

The talk is based on joint work with Chun-Hung Liu, Jean-Sebastien Sereni, Peter Whalen and Zelealem Yilma.

Lutz Warnke (Oxford): *Convergence of Achlioptas processes via differential equations with unique solutions*

In Achlioptas processes, starting from an empty graph, in each step two potential edges are chosen uniformly at random, and using some rule one of them is selected and added to the evolving graph. The evolution of the rescaled size of the largest component in such variations of the Erdős–Rényi random graph process has recently received considerable attention, in particular for the so-called product rule. In this work we establish the following result for rules such as the product rule: the limit of the rescaled size of the 'giant' component exists and is continuous provided that a certain system of differential equations has a unique solution. In fact, our result applies to a very large class of Achlioptas-like processes.

Our proof relies on a general idea which relates the evolution of stochastic processes to an associated system of differential equations. Provided that the latter has a unique solution, our approach shows that certain discrete quantities converge (after appropriate rescaling) to this solution.

Joint work with Oliver Riordan.

Yury Person (Berlin): *A randomized version of Ramsey's theorem and lower bounds on probability thresholds for Ramsey properties*

For every integer $r \geq 2$ we call a k-uniform hypergraph H on n vertices r-Ramsey-forcing if every r-edge-coloring of K_n contains a monochromatic copy of K_k whose vertices form a hyperedge in H. We determine the threshold for a random k-uniform hypergraph on n vertices and edge probability q to be r- Ramsey-forcing.

Further we discuss common features of the above problem with the more classical Ramsey's theorem for random (hyper-)graphs. There one is interested in p such that a.a.s. no matter how one colors the edges of the random (hyper-)graph $G(n, p)$ there will always be a monochromatic copy of F. We show the so-called 0-statement for a wide class of hypergraphs F.

This is joint work with Luca Gugelmann, Angelika Steger and Henning Thomas.

Heidi Gebauer (ETH Zürich): *The Local Lemma is Tight for SAT*

The (k, s)-SAT problem involves CNF formulas where each clause has k distinct literals and every variable occurs in at most s clauses. We investigate the maximum value $f(k)$ such that every instance of the $(k, f(k))$-SAT problem is a YES-instance. The determination of this extremal function is particularly important as it represents the value where the SAT problem exhibits its complexity hardness jump: The $(k, f(k) + 1)$-SAT problem is already NP-hard! The (lopsided) Local Lemma gives that $f(k) >= (2/e + o(1))2^k$. We will show that this bound is tight by constructing appropriate CNF formulas via special binary trees.

Anusch Taraz (Munich): *Maximum planar subgraphs in graphs near the minimum degree threshold $n/2$*

In this talk we investigate the size of a maximum planar subgraph in a graph with high minimum degree and show that there is a rapid change when the minimum degree is near $n/2$.

Denote by $\pi(n, d)$ the maximum number m such that every n-vertex graph G with minimum degree d contains a planar subgraph with m edges. While it is easy to see that $\pi(n, n/2) = 2n - 4$, Kühn, Osthus and Taraz showed that $\pi(n, n/2 + o(n)) = 3n - O(1)$. Trying to close the gap between these two statements, we will show in this talk that $\pi(n, n/2 + 1) \geq 2.25n$.

Joint work with Tomasz Łuczak and Andreas Würfl.

Mark Walters (Queen Mary): *Random Geometric Graphs*

Many natural random graph models involve some underlying spatial structure in their definition. We call such models Random Geometric Graphs. One example is to scatter n points at random in a square box and to join two if they are 'close'. Random graphs of this nature tend to be very different from Erdős-Rényi random graphs (for example cliques are much more common) and also very different from scale-free random graphs (vertices all have roughly the same degree).

In this talk we will discuss some of these Random Geometric Graph models, their similarities and differences and some common proof techniques.

Jan Volec (Prague): *Properties of cubic graphs with large girth and random cubic graphs*

We study asymptotic properties of large cubic graphs with large girth. Suppose G is such an n-vertex graph. We prove the following estimates: G contains an edge-cut of size $1.33008n$ and it has fractional cut covering number at most 1.12776. We show that G contains an independent set of size $0.4352n$ and it has fractional chromatic number at most 2.2978.

Our lower bounds on the size of maximum edge-cut and on the size of maximum independent set also apply to random cubic graphs. Specifically, a random n-vertex cubic graph asymptotically almost surely contains an edge-cut of size $1.33008n$ and an independent set of size $0.4352n$. (joint work with Daniel Král' and František Kardoš)

Mihyun Kang (Graz): *On the connectivity of random graphs from addable classes*

A class $\mathcal{G}$ of graphs is called *weakly addable* if for any $G \in \mathcal{G}$ and any two distinct components C_1 and C_2 in G, any graph that can be obtained by adding an edge between C_1 and C_2 is also in $\mathcal{G}$. McDiarmid, Steger and Welsh conjectured [Random graphs from planar and other addable classes. In Topics in discrete mathematics, volume 26 of Algorithms Combin., pages 231–246. Springer, 2006] that a random graph on n vertices from $\mathcal{G}$ is connected with probability at least $e^{-1/2} + o(1)$, as $n \to \infty$. We prove this conjecture under the additional assumption that $\mathcal{G}$ is *monotone*, i.e., if for any $G \in \mathcal{G}$ and any edge $e \in G$, the graph G without e is also in $\mathcal{G}$. (This is joint work with Konstantinos Panagiotou.)

Boris Bukh (Cambridge): *Generalized Erdős-Szekeres theorems*

We introduce a generalization of Erdős-Szekeres theorem, and show how it can be applied to construct a strengthening of weak epsilon-net. We also discuss the decision problem for Erdős-Szekeres-type for arbitrary semialgebraic predicates.

Tuesday

Alexandr Kostochka (Illinois): *$K_{s,t}$-minors in dense graphs and in $(s+t)$-chromatic graphs*

The goal of the talk is to refine two known results on the existence of $K_{s,t}$-minors in graphs. First we prove that if $(t/\log_2 t) \geq 1000s$, then every graph G with average degree at least $t + 8s\log_2 s$ has a $K^*_{s,t}$-minor, where $K^*_{s,t}$ is the graph obtained from $K_{s,t}$ by adding the edges of a complete graph on the first partite set. This result refines a result by Kühn and Osthus and is joint work with N. Prince.

It was proved earlier that for each s and

$$t > t_0(s) := \max\{4^{15s^2+s}, (240s\log_2 s)^{8s\log_2 s+1}\},$$

every graph with chromatic number $s+t$ has a $K^*_{s,t}$ minor. This result confirmed a special case of a conjecture by Woodall and Seymour. Using the first result, we show that the conclusion holds already for much smaller t, namely, for $t > C(s\log s)^3$.

Tobias Müller (CWI Amsterdam): *Line arrangements and geometric representations of graphs*

A dot product representation of a graph assigns to each vertex s a vector $v(s)$ in R^k in such a way that $v(s)^T v(t) > 1$ if and only st is an edge. Similarly, in a distance representation $|v(s) - v(t)| < 1$ if and only if st is an edge.

I will discuss the solution of some open problems by Spinrad, Breu & Kirkpatrick and others on these and related geometric representations of graphs. The proofs make use of a connection to oriented pseudoline arrangements.

Andrew Treglown (Prague): *Embedding spanning bipartite graphs of small bandwidth*

A graph H on n vertices has bandwidth at most b if there exists a labelling of the vertices of H by the numbers $1, \dots, n$ such that for every edge ij of H, $|i-j|$ is at most b. Boettcher, Schacht and Taraz gave a condition on the minimum degree of a graph G on n vertices that ensures G contains every r-chromatic graph H on n vertices of bounded degree and of bandwidth $o(n)$, thereby proving a conjecture of Bollobás and Komlós. We strengthen this result in the case when H is bipartite. Indeed, we give an essentially best-possible condition on the degree sequence of a graph G on n vertices that forces G to contain every bipartite graph H on n vertices of bounded degree and of bandwidth $o(n)$. This also implies an Ore-type result. In fact, we prove a much stronger result where the condition on G is relaxed to a certain robust expansion property. This is joint work with Fiachra Knox.

Hiep Hàn (Sao Paulo): *Turán problem for odd cycles in pseudorandom graphs*

In the last decades, much effort has been dedicated to transferring classical results in Extremal Combinatorics to sparse (pseudo-)random settings. While for the random case breakthroughs were recently achieved, transference in the pseudo-random setting remains wide open.

In this talk we shall put emphasis on Turan-type questions for graphs and give an almost best possible pseudorandomness condition for the Turan property for odd cycles.

This is a joint work with Elad Aigner-Horev and Mathias Schacht.

Wojciech Samotij (Cambridge): *Sum-free sets in Abelian groups*

A set A of elements of an (additively written) group G is sum-free if there are no x, y, z in A satisfying the equation $x + y = z$. Making such a definition leads to three natural questions: What is the size of the largest sum-free subset of G? How many sum free subsets does G contain? What is the structure of the largest/a typical sum-free subset of G? In the past 40 years, all three of the above questions have been answered fairly precisely in the case when G is Abelian. In this talk, we will first survey some of those answers and then present a few new results addressing the above questions in a more general setting. In particular, for every Abelian group G whose order is divisible by a prime congruent to 2 modulo 3, we will describe the structure of largest sum-free sets in a random binomial subset of G and the structure of a typical m-element sum-free subset of G. These new results are joint work with Noga Alon, Jozsef Balogh, and Robert Morris.

Graham Brightwell (LSE): *Random Methods in Partial Orders*

We give some examples of the use of random methods in proving extremal results in the theory of finite partial orders, highlighting some interesting problems that remain open.

Dan Hefetz (Birmingham): *Embedding spanning trees in random graphs near the connectivity threshold*

We prove that a given tree T on n vertices with bounded maximum degree is contained almost surely in the binomial random graph $G(n, (1+\varepsilon)logn/n)$ provided that T belongs to one of the following two classes: (1) T has linearly many leaves; (2) T has a path of linear length all of whose vertices have degree two in T.

Based on joint work with Michael Krivelevich and Tibor Szabó.

Béla Csaba (Birmingham): *Embedding spanning trees*

There exists three positive constants c, K and n_0 such that the following holds. Let $n \geq n_0$, and assume that T is a tree on n vertices with maximum degree $m \leq cn/\log n$. If G is a graph on n vertices having minimum degree $\delta(G) \geq n/2 + Km \log n$ then $T \subset G$. Joint work with A. Jamshed and E. Szemerédi.

Jozef Skokan (LSE): *Multipartite Version of the Alon-Yuster Theorem*

In this talk, we prove the asymptotic multipartite version of the Alon-Yuster theorem. That is, if $k \geq 3$ is an integer, H is a k-colorable graph and $\gamma > 0$ is fixed, then for sufficiently large n and for every balanced k-partite graph G on kn vertices with each of its corresponding $\binom{k}{2}$ bipartite subgraphs having minimum degree at least $(k-1)n/k + \gamma n$, the graph G has a subgraph consisting of $\lfloor n/|V(H)| \rfloor$ vertex-disjoint copies of H. Joint work with Ryan Martin.

Wednesday

Demetres Christofides (Queen Mary): *Matrix-valued random variables and applications*

In this talk we will present an Azuma-Hoeffding type inequality for matrix-valued random variables and discuss some of its applications in graph theory and in particular in the theory of random Cayley graphs.

Konstantinos Panagiotou (LMU München): *Random Planar Graphs*

The Erdős-Rényi model, which was introduced in the 60's, has led to numerous beautiful and deep theorems about the structural characteristics of a 'typical' graph. A key feature that is used in basically all proofs is that edges appear independently in such graphs. This situation changes dramatically if one considers graph classes with structural side constraints. For example, in a random planar graph, i.e., a graph drawn uniformly at random from the class of all labelled planar graphs with a given number of vertices, the edges are far from being independent. In this talk we survey progress on the study of properties of random planar graphs and the methods that were used and developed in order to achieve this progress.

John Talbot (UCL): *Extremal problems for the hypercube*

There are a number of very natural Turán and Ramsey problems for the hypercube. For example, an old question of Erdős asks whether every subgraph of the n-cube with (significantly) more than a half of the edges contains a 4-cycle?

I will survey some of the known results and open problems in this area. New results I hope to mention include Ramsey results for subsets of vertices of the hypercube (joint work with John Goldwasser) as well as Turán results for both edges and vertices of the hypercube (due to Rahil Baber).

Klas Markström (Umea): *Random graphs: ordinary, regular and cayley*

I will discuss some of the similarities and differences between the ordinary random graph, the random regular graph and random cayley graphs of different groups. For high densities these models are locally very similar but global properties set them apart to varying degrees. This is based on joint work with Demetres Christofides.

Frédéric Havet (INRIA): *Decomposing a graph into clique-like sets*

B. Reed showed that one can partition the vertex set of a graph G of maximum degree Δ into sets $D_1, \dots, D_l$ and S where the D_i are *clique-like* sets and all the vertices in S are *sparse*. A set D is *clique-like* in that:

(i) it has about Δ vertices,
(ii) most of the pairs of vertices within it are adjacent,
(iii) there are only few edges between D and $G - D$,
(iv) a vertex is in D if and only if most of its neighbours are.

A vertex is *sparse* if the the subgraph induced by its neighbourhood contains fewer than t edges for some fixed threshold t.

This decomposition lemma has been a very useful tool to prove several graph colouring theorems :

- Molloy and Reed used it to prove the following strengthening of Brooks' Theorem: for all sufficiently large Δ and k such that $k^2 + k < \Delta$, there exists N such that, if G has maximum degree Δ and $\chi(G) > \Delta + 1 - k$, then there exists a subgraph H of G with at most N vertices such that $\chi(H) > \Delta + 1 - k$.
- The same authors also used it to prove that for some absolute constant C, every graph of maximum degree Δ has a $(\Delta + C)$-total-colouring.
- It is a key ingredient of Havet, Reed and Sereni proof that for sufficiently large Δ every graph with maximum degree Δ has an $L(2, 1)$-labelling with $\Delta^2 + 1$ colours.

In this talk, we will precisely state the decomposition lemma and explain how such a tool may be used.

Oleg Pikhurko (Warwick): *Asymptotic Structure of Graphs with the Minimum Number of Triangles*

We describe the structure of graphs, modulo changing $o(1)$-fraction of edges, that have the minimum number of triangles given their order and size. Joint work with Alexander Razborov.

Peter Allen (LSE): *Tight Hamilton cycles in random hypergraphs*

Results on Hamiltonicity of graphs are classical: Dirac's theorem gives the minimum degree guaranteeing Hamiltonicity, while in random graphs Bollobas, and Bollobas, Fenner and Frieze (respectively) established very precisely the threshold edge probability at which a random graph is almost surely Hamiltonian, and found an algorithm which returns a Hamilton cycle at the same probability.

In hypregraphs, however, until recently very little has been known. With the 'absorbing method' it has recently been possible to determine minimum degree thresholds (for various notions of minimum degree) for Hamiltonicity (depending again on the definition): but this method does not seem to work on sparse graphs. Nevertheless Frieze, and Frieze and Dudek, have been able to determine thresholds for the appearance of various flavours of Hamilton cycle in hypergraphs using the second moment method. In this talk we introduce a combinatorial method, replacing the absorbing method, which we apply to find tight Hamilton cycles in random hypergraphs at an edge probability of $n^{-1+\varepsilon}$ for each $\varepsilon > 0$. Our method is algorithmic.

This is joint work with Julia Böttcher, Yoshiharu Kohayakawa and Yury Person.

Julia Böttcher (LSE): *Powers of Hamilton cycles in pseudo-random graphs*

We say that a graph G is (ϵ, p, s, t)-pseudo-random if for any pair X, Y of disjoint vertex sets in G with $|X| > \epsilon p^s n$ and $|Y| > \epsilon p^t n$ we have that $e(X, Y)$ differs from the expected value $p|X||Y|$ by at most $\epsilon p|X||Y|$.

We show that an $(\epsilon, p, 1, 2)$-pseudo-random graph on n vertices with minimum degree at least $pn/2$ contains the square of a Hamilton cycle if $p^2 n > C$ for some constant C. In particular this implies that (n, d, λ)-graphs with $\lambda \ll d^{5/2} n^{-3/2}$, which exist provided $d > n^{3/4}$, contain the square of a Hamilton cycle; and thus a triangle factor if n is a multiple of 3. This improves on a result of Krivelevich, Sudakov and Szabó.

We also extend our result to higher powers of Hamilton cycles: For $k > 2$ we show that $(\epsilon, p, k-1, 2k-1)$-pseudo-random graphs on n vertices with minimum degree at least $pn/2$ contain the k-th power of a Hamilton cycle if $p^{2k-1} n > C$.

Joint work with Peter Allen, Hiep Han, Yoshiharu Kohayakawa, Yury Person.

Stefanie Gerke (Royal Holloway): *Factors in Random Graphs*

An H-factor of a graph G is a collection of disjoint copies of H in G that covers every vertex of G. In a seminal paper Johansson, Kahn and Vu determined the threshold $th(H)$ of the property that $G_{n,p}$ contains an H-factor if H is strictly balanced. In this talk we discuss the main ideas of this paper and show how one can use the methods to find the thresholds $th(H)$ for graphs H that are non-vertex balanced. This talk is based on joint work with Andrew McDowell.

Thursday

David Conlon (Oxford): *Graph regularity and removal lemmas*

Szemerédi's regularity lemma states that every large graph may be partitioned into a small number of parts so that the bipartite graph between almost all pairs of parts is random-like. One of the most important applications of this theorem is the graph removal lemma, which roughly says that every graph with few copies of a fixed graph H can be made H-free by removing few edges. In this talk, we will discuss recent progress on bounds for these theorems and for several important variants. This is joint work with Jacob Fox.

Jan Hladký (Warwick): *Loebl-Komlós-Sós Conjecture, and structure of sparse graphs*

A graph with minimum degree k contains each tree of order $k + 1$ as a subgraph. Erdős and Sós (ES), and Loebl, Komlós and Sós (LKS) conjectured that the assertion remains true even when the minimum degree is replaced by weaker notion of average degree and median degree, respectively.

I will talk about a general graph decomposition technique which we used with J. Komlós, D. Piguet, M. Simonovits, M. Stein and E. Szemerédi to prove an asymptotic version of the LKS Conjecture. This method can be viewed as an extension of the Szemerédi Regularity Lemma to the sparse setting. We borrow heavily from previous unpublished work of M. Ajtai, J. Komlós, M. Simonovits, and E. Szemerédi on the ES Conjecture.

Viresh Patel (Durham): *Graph Partitioning Problems*

It is well known that the vertex set of every graph can be partitioned into two sets such that at least half the edges of the graph cross the partition. The constant $1/2$ is best possible because of the complete graph. (More generally a graph can be partitioned into k parts such that the proportion of crossing edges is at least $(k-1)/k$, and again $(k-1)/k$ is best possible due to the complete graph.)

In a sequence of seminal papers, Bollobás and Scott investigated extensions to this by considering what they called judicious partition problems. In such problems, one wishes to study extremal graph-partition questions where two (or more) parameters are to be optimized simultaneously. One such question is the following: given two graphs G and H on the same vertex set V, does there exist a partition of V so that at least half the edges of G AND half the edges of H cross the partition? We discuss some of the probabilistic techniques that can be used to approach such problems.

Imre Leader (Cambridge): *Geodesics in Subgraphs of the Cube*

It is easy to see that a subgraph of the cube of average degree k contains a geodesic of length $k/2$. Our main result is that in fact it must contain a geodesic of length k, which is best possible. This is related to Norine's antipodal colouring conjecture. (joint work with Eoin Long)