Joint Birmingham–Warwick Combinatorics seminar, March 6th

Short talk 1: Lecture Room C, Watson Building 1.30–2pm
Deciding the Bell number for hereditary graph classes
Jan Foniok (University of Warwick)

The speed of a hereditary graph class is the number of (labelled) graphs on $\{1, 2, \ldots, n\}$ in the class, as a function of n. It is known that not every function can be obtained as the speed of some such class, e.g., if the speed grows faster than any polynomial, than it is at least exponential.

Another such jump was identified by Balogh, Bollobás and Weinreich (2005): if the speed is at least $n^{(1-o(1))n}$, then it is bounded from below by the nth Bell number (the number of partitions of an n-element set). We study the computational problem to decide for a given hereditary class whether its speed is below or above* the Bell number. We provide an algorithm that solves this problem for classes described by a finite list of forbidden induced subgraphs. It is based on a characterisation of minimal classes with speed above* the Bell number.

The talk is based on joint work with A. Atminas, A. Collins and V. Lozin.

* By "above" I mean "greater than or equal to".

Short talk 2: Lecture Room B, Watson Building 2–2.30pm
Proof of a tournament partition conjecture and an application to 1-factors with prescribed cycle lengths
Tim Townsend (University of Birmingham)

In 1982 Thomassen asked whether there exists an integer $f(k, t)$ such that every strongly $f(k, t)$-connected tournament T admits a partition of its vertex set into t vertex classes $V_1, ..., V_t$ such that for all i the subtournament $T[V_i]$ induced on T by V_i is strongly k-connected. Our main result implies an affirmative answer to this question. In particular we show that $f(k, t) = O(k^7 t^4)$ suffices. As another application of our main result we give an affirmative answer to a question of Song as to whether, for any integer t, there exists an integer $h(t)$ such that every strongly $h(t)$-connected tournament has a 1-factor consisting of t vertex-disjoint cycles of prescribed lengths. We show that $h(t) = O(t^5)$ suffices.

The talk is based on joint work with Daniela Kühn and Deryk Osthus.

Coffee break 2.30-3.00pm

Main seminar: Lecture Room B, Watson Building 3–3.50pm
Discrepancy of graphs and hypergraphs
Alex Scott (University of Oxford)

How uniformly is it possible to distribute edges in a graph? For instance, is there a graph of density 1/2 in which every induced subgraph has approximately the same number of edges as nonedges?

Erdős and Spencer showed in 1971 that every graph on n vertices has an induced subgraph in which the numbers of edges and nonedges differ by at least $cn^{3/2}$ (and gave a similar result for hypergraphs). Erdős, Goldberg, Pach and Spencer subsequently proved a weighted extension of this for graphs with density p.

We shall discuss generalizations of these results and related questions involving intersections of pairs of graphs or hypergraphs. Joint work with Béla Bollobás.