What is Ramsey-equivalent to the clique?

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A graph G is Ramsey for H if every two-colouring of the edges of G contains a monochromatic copy of H. Two graphs H and H' are Ramsey-equivalent if every graph G is Ramsey for H if and only if it is Ramsey for H'. We study the problem of determining which graphs are Ramsey-equivalent to the complete graph K_k. A famous theorem of Folkman implies that any graph H which is Ramsey-equivalent to K_k must contain K_k. Recently, Fox, Grinshpun, Person, Szabó and the speaker proved that the only connected graph which is Ramsey-equivalent to K_k is itself. This gives a negative answer to the question of Szabó, Zumstein, and Zürcher on whether K_k is Ramsey-equivalent to $K_k.K_2$, the graph on $k+1$ vertices consisting of K_k with a pendant edge. In fact, a stronger result is true. A graph G is Ramsey minimal for a graph H if it is Ramsey for H but no proper subgraph of G is Ramsey for H. Let $s(H)$ be the smallest minimum degree over all Ramsey minimal graphs for H. The study of $s(H)$ was introduced by Burr, Erdős, and Lovász, where they show that $s(K_k) = (k-1)^2$. In the aforementioned work, we proved that $s(K_k.K_2) = k-1$, and hence K_k and $K_k.K_2$ are not Ramsey-equivalent. We also addressed the question of which non-connected graphs are Ramsey-equivalent to K_k. Let $f(k,t)$ be the maximum f such that the graph $H = K_k + fK_t$, consisting of K_k and f disjoint copies of K_t, is Ramsey-equivalent to K_k. Szabó, Zumstein, and Zürcher gave a lower bound on $f(k,t)$. We proved an upper bound on $f(k,t)$ which is roughly within a factor 2 of the lower bound.