Probabilistic Intuition in Waiter–Client and Client–Waiter games

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For a finite set X, a family of sets $\mathcal{F} \subseteq 2^X$ and a positive integer q, we consider two types of two player, perfect information games with no chance moves. In each round of the $(1 : q)$ Waiter–Client game $(X, \mathcal{F})$, the first player, called Waiter, offers the second player, called Client, $q + 1$ elements of the board X which have not been offered previously. Client then claims one of these elements and the remaining q elements go back to Waiter. Waiter wins this game if, by the time every element of X has been claimed by some player, Client has claimed all elements of some $A \in \mathcal{F}$; otherwise Client is the winner. Client–Waiter games are defined analogously, the main difference being that Client wins the game if he manages to claim all elements of some $A \in \mathcal{F}$ and Waiter wins otherwise. In this talk, we will look at the Waiter–Client and Client–Waiter versions of the non–planarity, K_t–minor and non–k–colourability games. For each such game, we give a fairly precise estimate of the unique integer q at which the outcome of the game changes from Client's win to Waiter's win. We also discuss the relationship between our results, random graphs, and the corresponding Maker–Breaker and Avoider–Enforcer games.

This is joint work with Dan Hefetz (University of Birmingham) and Michael Krivelevich (Tel Aviv University).