Heuristic Optimisation

Problem sheet 4

Brief solutions

1. For the 8-puzzle problem, heuristics can be derived by relaxing one or both of the constraints:

   - A tile can only move from square A to square B if A is adjacent to B.
   - A tile can only move from square A to square B if B is empty.

   "Tiles in wrong position" is obtained by relaxing both constraints. The heuristic which allows moves to empty squares only is obtained by relaxing the first constraint. Give an example where this heuristic is better than the tiles in wrong position (i.e., gives a better estimate). Give an example where this heuristic is better than the Manhattan distance (which is obtained by relaxing the second constraint only).

Solution

Let h_1 - the new heuristic

h_2 - tiles in wrong position

h_3 - Manhattan distance

(a)

Start state

1	2	3
4		8
7	6	5

Goal state

1	2	3
8		4
7	6	5

$h_1(\text{start}) = 3$ — h_1 is better than h_2 for this case

$h_2(\text{start}) = 2$

(b)

Start state

1	3	2
8		4
7	6	5

Goal state

1	2	3
8		4
7	6	5

$h_1(\text{start}) = 3$ — h_1 is better than h_2

$h_2(\text{start}) = 2$ — h_1 is better than h_3

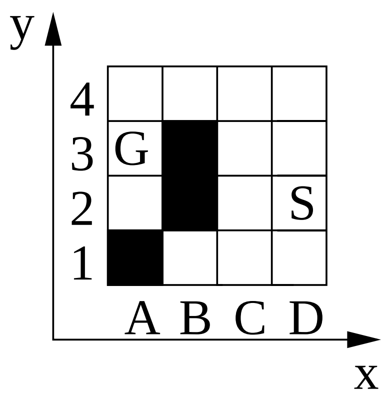
$h_3(\text{start}) = 2$

2. Three admissible heuristics h_1, h_2, h_3 are given for an optimisation problem to be solved using A*. None of the heuristics is better than (i.e., dominates) the others. Explain how a new heuristic could be built based on these three heuristics so that the new heuristic is better than all three given heuristics.

Solution

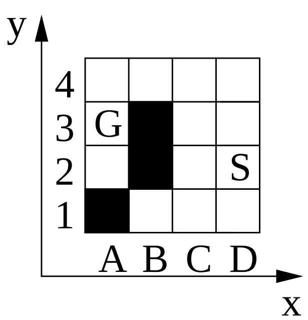
$\max(h_1, h_2, h_3)$

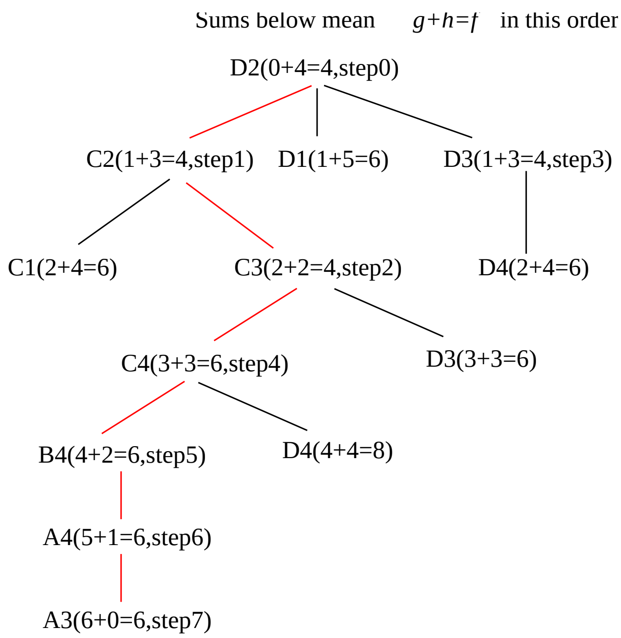
3. Consider the maze given below.



The black squares are obstacles. The problem is to find the shortest path from the start position S to the goal position G. Describe an appropriate heuristic for A* search and show the search tree generated by A* search.

Solution





4. Consider the following Boolean Satisfiability problem:

$$F(x_1, x_2, x_3, x_4, x_5) = (\bar{x}_1 \bigvee \bar{x}_2) \bigwedge (x_1 \bigvee \bar{x}_2 \bigvee \bar{x}_4) \bigwedge (x_1 \bigvee x_2 \bigvee \bar{x}_3) \bigwedge (\bar{x}_1 \bigvee \bar{x}_5).$$

Starting from the point $(x_1, x_2, x_3, x_4, x_5) = (1, 1, 1, 1, 1)$ apply the tabu search method with memory of 3 steps to find a solution. Show the memory at each step.

Solution:

	1	2	3	4	
	0	1	1	0	
flip 1	1	0	1	1	+1
flip 2	1	1	1	0	+1
flip 3	0	1	1	0	0
flip 4	0	1	1	0	0
flip 5	0	1	1	1	+1

Flip x_1. The memory vector becomes $(3, 0, 0, 0, 0)$. The variable x_1 cannot be flipped for 3 steps. We get $(x_1, x_2, x_3, x_4, x_5) = (0, 1, 1, 1, 1)$.

	1	2	3	4	
	1	0	1	1	
flip 2	1	1	0	1	0
flip 3	1	0	1	1	0
flip 4	1	1	1	1	+1
flip 5	1	0	1	1	0

We should flip x_4. The memory vector becomes $(2, 0, 0, 3, 0)$. The variable x_1 cannot be flipped for 2 steps and the variable x_4 cannot be flipped for three steps. The point $(x_1, x_2, x_3, x_4, x_5) = (0, 1, 1, 0, 1)$ is a solution.

5. Suppose you use tabu search for the Boolean satisfiability problem with 8 variables. The initial assignment is $\mathbf{x} = (0, 0, 1, 1, 0, 1, 0, 1)$. After 4 iterations the recency-based memory is

4	1	0	2	0	0	3	0

.

What is the value of $\mathbf{x}$ after 4 iterations and why?

Solution:

The given memory shows that the memory length is 4. It can be observed that during the 4 iterations variables x_3, x_5, x_6 and x_8 have not been flipped (otherwise it is not possible to get 0 in the 3rd, 5th, 6th and 8th cell of the memory after 4 iterations, because the memory length is 4) and variables x_1, x_2, x_4, x_7 have been flipped (because the 1st, 2nd, 4th and 7th memory cells are nonzero), and moreover they have been flipped exactly once (note that it is not possible to flip a variable more than once in 4 iterations because the memory length is 4). Therefore after 4 iterations $\mathbf{x} = (1, 1, 1, 0, 0, 1, 1, 1)$.