Heuristic Optimisation

Problem sheet 4

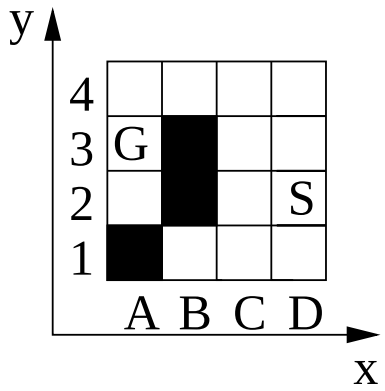
1. For the 8-puzzle problem, heuristics can be derived by relaxing one or both of the constraints:

   - A tile can only move from square A to square B if A is adjacent to B.
   - A tile can only move from square A to square B if B is empty.

   "Tiles in wrong position" is obtained by relaxing both constraints. The heuristic which allows moves to empty squares only is obtained by relaxing the first constraint. Give an example where this heuristic is better than the tiles in wrong position (i.e., gives a better estimate). Give an example where this heuristic is better than the Manhattan distance (which is obtained by relaxing the second constraint only).

2. Three admissible heuristics h_1, h_2, h_3 are given for an optimisation problem to be solved using A*. None of the heuristics is better than (i.e., dominates) the others. Explain how a new heuristic could be built based on these three heuristics so that the new heuristic is better than all three given heuristics.

3. Consider the maze given below.



   The black squares are obstacles. The problem is to find the shortest path from the start position S to the goal position G. Describe an appropriate heuristic for A* search and show the search tree generated by A* search.

4. Consider the following Boolean Satisfiability problem:

$$F(x_1, x_2, x_3, x_4, x_5) = (\bar{x}_1 \bigvee \bar{x}_2) \bigwedge (x_1 \bigvee \bar{x}_2 \bigvee \bar{x}_4) \bigwedge (x_1 \bigvee x_2 \bigvee \bar{x}_3) \bigwedge (\bar{x}_1 \bigvee \bar{x}_5).$$

   Starting from the point $(x_1, x_2, x_3, x_4, x_5) = (1, 1, 1, 1, 1)$ apply the tabu search method with memory of 3 steps to find a solution. Show the memory at each step.

5. Suppose you use tabu search for the Boolean satisfiability problem with 8 variables. The initial assignment is $\mathbf{x} = (0, 0, 1, 1, 0, 1, 0, 1)$. After 4 iterations the recency-based memory is

4	1	0	2	0	0	3	0

   What is the value of $\mathbf{x}$ after 4 iterations and why?