Heuristic Optimisation

Problem sheet 2

1. Consider the following 6 dimensional SAT problem.

$$F(x_1, x_2, x_3, x_4, x_5, x_6) = (x_1 \bigvee \overline{x}_2 \bigvee x_3) \bigwedge (\overline{x}_2 \bigvee \overline{x}_6) \bigwedge (x_3 \bigvee x_6)$$

Enumerate the search space for this problem. Construct the truth table and find the solutions to the SAT problem, if there are any.

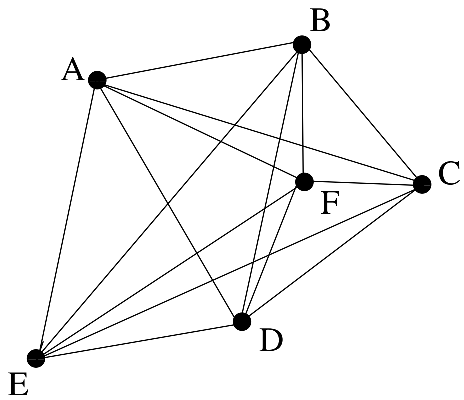
2. Consider the following 4 dimensional SAT problem.

$$F(x_1, x_2, x_3, x_4) = (x_1 \bigvee \overline{x}_2 \bigvee x_3) \bigwedge (x_2 \bigvee \overline{x}_3) \bigwedge (x_3 \bigvee x_4) \bigwedge x_1$$

Apply the GSAT local search algorithm from starting point $x_1 = 0$, $x_2 = 0$, $x_3 = 0$, $x_4 = 0$. Can you find a solution if $MAX\_FLIPS = 2$?

3. Consider a symmetric TSP problem with 6 cities A, B, C, D, E, F. The distances between cities are as follows:

City	City	Distance
A	B	1
A	C	3
A	D	3
A	E	4
A	F	2
B	C	2
B	D	4
B	E	5
B	F	1
C	D	2
C	E	3
C	F	1
D	E	1
D	F	2
E	F	2



(a) Apply the nearest neighbour heuristic optimisation method starting from city A and show the solution that you find.

(b) Apply the nearest neighbour heuristic optimisation method starting from city C and show the solution that you find.

(c) Explain whether differences can occur when applying the nearest neighbour method from different starting points.

(d) Apply a different greedy heuristic for solving this TSP problem. Is your solution better or worse than the ones found in (a) and (b)?

4. Consider the 0/1 knapsack problem:

   A set of N items, each having a value v_i and a weight w_i are given. There is a bag of finite capacity K (maximum weight it can hold). The problem is to fill the bag with items (without exceeding its capacity), so that the total value of items in the bag is maximised. You can either put an item in the bag as a whole or leave it out, it is not possible to use parts of an item.

   (a) Describe a greedy algorithm for this problem.

   (b) For the particular case:
   $N = 5$
   $v_1 = 55,\ w_1 = 50$
   $v_2 = 50,\ w_2 = 24$
   $v_3 = 32,\ w_3 = 30$
   $v_4 = 24,\ w_4 = 20$
   $v_5 = 20,\ w_5 = 10$
   $K = 100$
   would the greedy algorithm find the optimal solution?

5. Consider the following Boolean Satisfiability problem:

$$F(x_1, x_2, x_3, x_4, x_5) = (\bar{x}_1 \bigvee \bar{x}_2) \bigwedge (x_1 \bigvee \bar{x}_2 \bigvee \bar{x}_4) \bigwedge (x_1 \bigvee x_2 \bigvee \bar{x}_3) \bigwedge (\bar{x}_1 \bigvee \bar{x}_5).$$

   Apply a greedy algorithm to solve this problem. Describe each step.