## Heuristic Optimisation

## Problem sheet 1

1. Mr. Smith and his wife invited four other couples for a party. When everyone arrived, some of the people in the room shook hands with some of the others. Of course, nobody shook hands with their spouse and nobody shook hands with the same person twice.

After that Mr. Smith asked everyone how many times they shook someone's hand. He received different answers from everybody.

How many times did Mrs. Smith shake someone's hand?
2. Discuss possible representations, size of search space, and possible evaluation functions for Rubik's cube. You may want to check a Rubik's cube application on your smart phone or an online virtual cube, such as

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http://www.speedcubing.com/CubeSolver/CubeSolver.html
http://www.mathplayground.com/rubikscube.html
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and play a little bit with it before thinking on this.
3. Consider the maze given below.


The black squares are obstacles. The objective is to get from the start node S to the goal node G , by moving horizontally and vertically. Use hill-climbing to solve this problem.
4. Consider the function $f(x, y)=x^{2}+x y+y^{2}+3 x+4 y$ defined on points of integer coordinates in the box $[-10,10] \times[-10,10]$. Use the hill-climbing algorithm to find the function's minimum value on this box. First define the search space, then the neighbourhood. Try to find the minimum using the starting point $(x, y)=(0,0)$.
5. Consider the following 6 dimensional SAT problem.

$$
\left.F\left(x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}\right)=\left(x_{1} \bigvee \bar{x}_{2} \bigvee x_{3}\right) \bigwedge\left(\bar{x}_{2} \bigvee \bar{x}_{6}\right) \bigwedge\left(x_{3} \bigvee x_{6}\right)\right)
$$

(a) Consider the incomplete solutions given by the binary strings of length 6 that start with 01. This property determines a subspace of the search space. Does there exist a solution of the problem which shares this property? If yes, then what should the value of $x_{3}$ and $x_{6}$ be for such a solution?
(b) Consider the incomplete solutions given by the binary strings ${ }^{* *} 0^{* *} 1$ (i.e, binary strings of length 6 with $x_{3}=0, x_{6}=1$ ). This property determines a subspace of the search space. Does there exist a solution of the problem which shares this property? If yes, then what should the value of $x_{2}$ be for such a solution?
(c) Consider the incomplete solutions given by the binary strings * $1 * 011$. This property determines a subspace of the search space. Determine all solutions of the problem which share this property.

