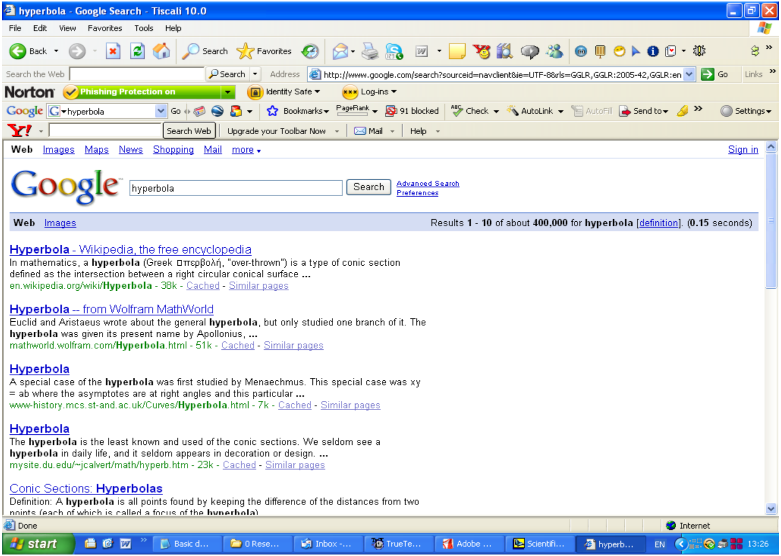
Google: An Application of Linear Algebra (The mathematics of a great success)

Peter Butkovic


hyperbola - Google Search - Tiscali 10.0
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Search the Web Search Address http://www.google.com/search?sourceid=navclient&ie=UTF-8&rls=GGLR,GGLR:2005-42,GGLR:en Go Links
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Web Images Results 1 - 10 of about 400,000 for hyperbola [definition]. (0.15 seconds)
Hyperbola - Wikipedia, the free encyclopedia
In mathematics, a hyperbola (Greek ὑπερβολή, "over-thrown") is a type of conic section defined as the intersection between a right circular conical surface ...
en.wikipedia.org/wiki/Hyperbola - 38k - Cached - Similar pages
Hyperbola -- from Wolfram MathWorld
Euclid and Aristaeus wrote about the general hyperbola, but only studied one branch of it. The hyperbola was given its present name by Apollonius, ...
mathworld.wolfram.com/Hyperbola.html - 51k - Cached - Similar pages
Hyperbola
A special case of the hyperbola was first studied by Menaechmus. This special case was xy = ab where the asymptotes are at right angles and this particular ...
www-history.mcs.st-and.ac.uk/Curves/Hyperbola.html - 7k - Cached - Similar pages
Hyperbola
The hyperbola is the least known and used of the conic sections. We seldom see a hyperbola in daily life, and it seldom appears in decoration or design. ...
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Conic Sections: Hyperbolas
Definition: A hyperbola is all points found by keeping the difference of the distances from two
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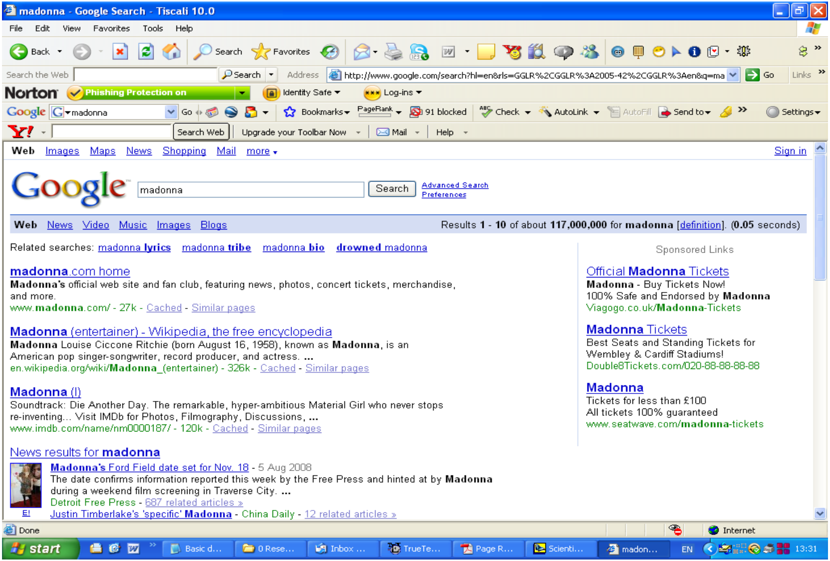
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Madonna's official web site and fan club, featuring news, photos, concert tickets, merchandise, and more.
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Madonna (entertainer) - Wikipedia, the free encyclopedia
Madonna Louise Ciccone Ritchie (born August 16, 1958), known as Madonna, is an American pop singer-songwriter, record producer, and actress. ...
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Madonna (I)
Soundtrack: Die Another Day. The remarkable, hyper-ambitious Material Girl who never stops re-inventing... Visit IMDb for Photos, Filmography, Discussions, ...
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News results for madonna
Madonna's Ford Field date set for Nov. 18 - 5 Aug 2008
The date confirms information reported this week by the Free Press and hinted at by Madonna during a weekend film screening in Traverse City. ...
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Justin Timberlake's 'specific' Madonna - China Daily - 12 related articles »
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Madonna
Tickets for less than £100
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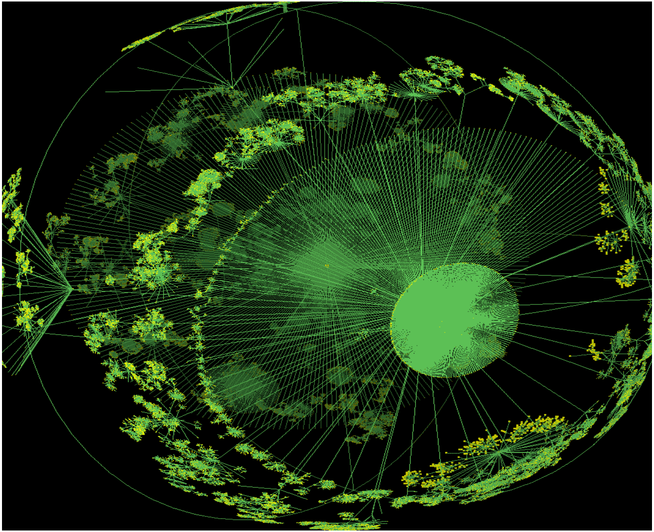
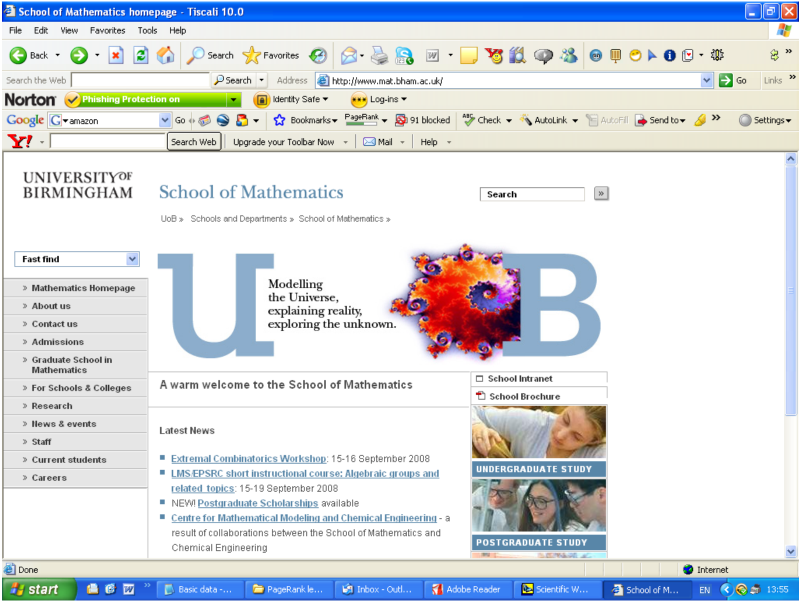


Hyun's Map of the Web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http://www.mat.bham.ac.uk/
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Admissions
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Research
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Staff
Current students
Careers
Modelling the Universe, explaining reality, exploring the unknown.
A warm welcome to the School of Mathematics
School Intranet
School Brochure
Latest News
Extremal Combinatorics Workshop: 15-16 September 2008
LMS/EPSRC short instructional course: Algebraic groups and related topics: 15-19 September 2008
NEW! Postgraduate Scholarships available
Centre for Mathematical Modeling and Chemical Engineering - a result of collaborations between the School of Mathematics and Chemical Engineering
UNDERGRADUATE STUDY
POSTGRADUATE STUDY
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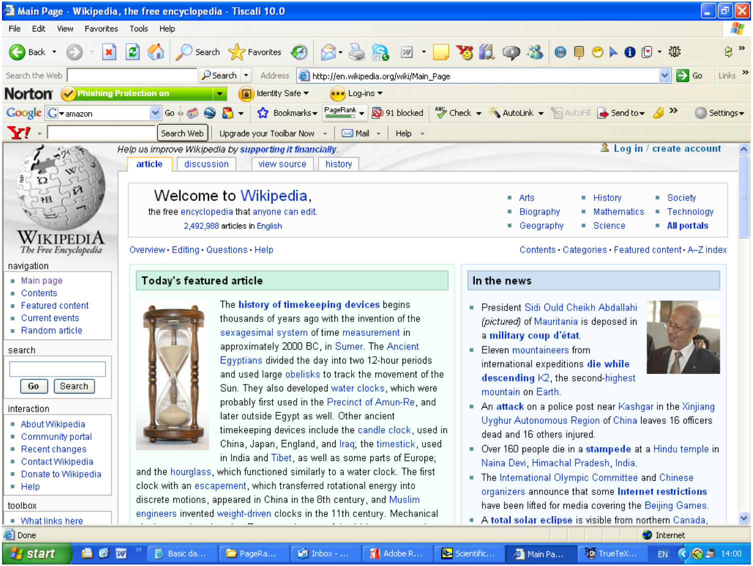
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Biography Mathematics Technology
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Today's featured article
The history of timekeeping devices begins thousands of years ago with the invention of the sexagesimal system of time measurement in approximately 2000 BC, in Sumer. The Ancient Egyptians divided the day into two 12-hour periods and used large obelisks to track the movement of the Sun. They also developed water clocks, which were probably first used in the Precinct of Amun-Re, and later outside Egypt as well. Other ancient timekeeping devices include the candle clock, used in China, Japan, England, and Iraq; the timestick, used in India and Tibet, as well as some parts of Europe; and the hourglass, which functioned similarly to a water clock. The first clock with an escapement, which transferred rotational energy into discrete motions, appeared in China in the 8th century, and Muslim engineers invented weight-driven clocks in the 11th century. Mechanical
In the news
President Sidi Ould Cheikh Abdallahi (pictured) of Mauritania is deposed in a military coup d'état.
Eleven mountaineers from international expeditions die while descending K2, the second-highest mountain on Earth.
An attack on a police post near Kashgar in the Xinjiang Uyghur Autonomous Region of China leaves 16 officers dead and 16 others injured.
Over 160 people die in a stampede at a Hindu temple in Naina Devi, Himachal Pradesh, India.
The International Olympic Committee and Chinese organizers announce that some Internet restrictions have been lifted for media covering the Beijing Games.
A total solar eclipse is visible from northern Canada,
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- Founders of Google: Larry Page and Sergey Brin

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- 1998: Google incorporates as a company (initial investment: \$1.1million) and files patent for PageRank

- 2000: Selling advertisements began

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- 2008: Google uses 450,000 servers in 25 locations around the world to index $>$ 8 billion websites

KEY IDEAS

- Each webpage is assigned a measure of importance ... PageRank

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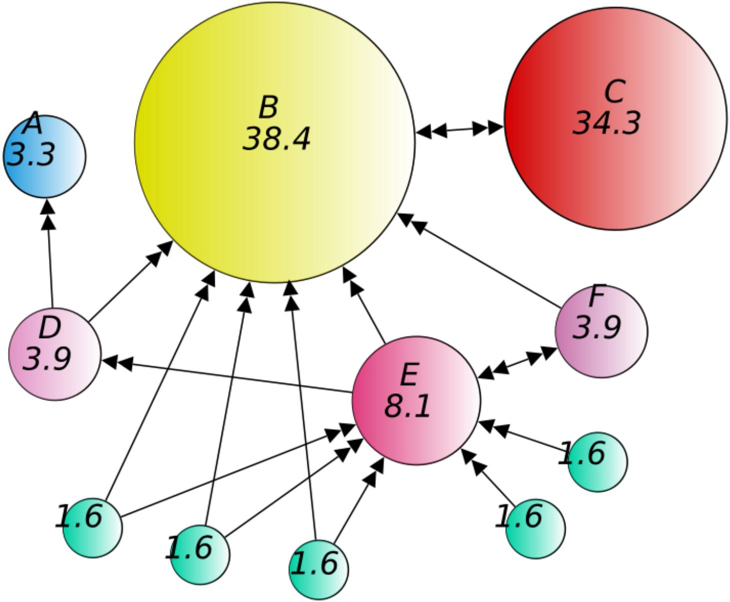
- Each webpage is assigned a measure of importance ... PageRank
- A hyperlink is a "recommendation": A webpage is important if other webpages point to it (via hyperlinks)
- The PageRank of the recommender matters (the higher the better)
- The generosity of the recommender matters (the higher the worse)


A
3.3
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C
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D
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E
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F
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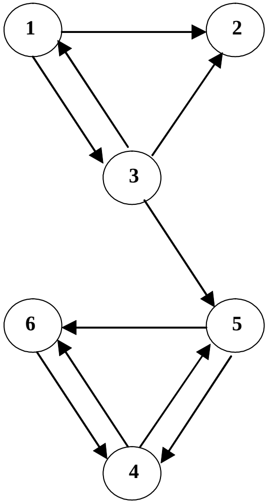
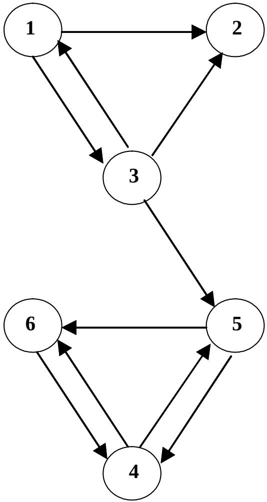
A.N.Langville and C.D.Meyer: Google's PageRank and Beyond (PUP 2006)



- $r(P_1) = \frac{1}{3} r(P_3)$

Node 2 - *dangling*

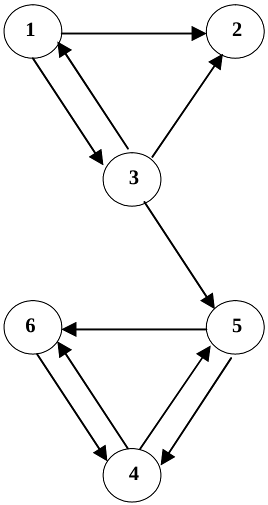
A.N.Langville and C.D.Meyer: *Google's PageRank and Beyond* (PUP 2006)



- $r(P_1) = \frac{1}{3} r(P_3)$
- $r(P_2) = \frac{1}{2} r(P_1) + \frac{1}{3} r(P_3)$

Node 2 - *dangling*

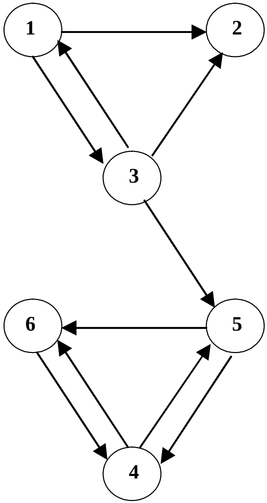
A.N.Langville and C.D.Meyer: Google's PageRank and Beyond (PUP 2006)



- $r(P_1) = \frac{1}{3} r(P_3)$
- $r(P_2) = \frac{1}{2} r(P_1) + \frac{1}{3} r(P_3)$
- $r(P_3) = \frac{1}{2} r(P_1)$

Node 2 - *dangling*

A.N.Langville and C.D.Meyer: Google's PageRank and Beyond (PUP 2006)



- $r(P_1) = \frac{1}{3} r(P_3)$
- $r(P_2) = \frac{1}{2} r(P_1) + \frac{1}{3} r(P_3)$
- $r(P_3) = \frac{1}{2} r(P_1)$
- $r(P_4) = \frac{1}{2} r(P_5) + r(P_6)$

Node 2 - *dangling*

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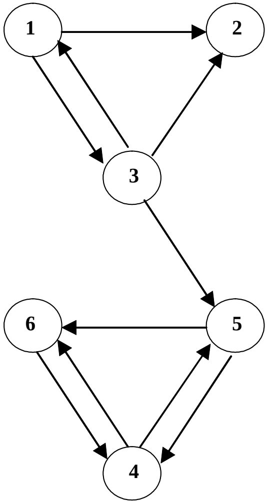

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- $r(P_3) = \frac{1}{2} r(P_1)$
- $r(P_4) = \frac{1}{2} r(P_5) + r(P_6)$
- $r(P_5) = \frac{1}{3} r(P_3) + \frac{1}{2} r(P_4)$

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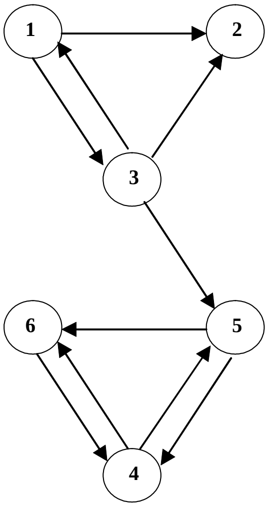

- $r(P_1) = \frac{1}{3} r(P_3)$
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- $r(P_3) = \frac{1}{2} r(P_1)$
- $r(P_4) = \frac{1}{2} r(P_5) + r(P_6)$
- $r(P_5) = \frac{1}{3} r(P_3) + \frac{1}{2} r(P_4)$
- $r(P_6) = \frac{1}{2} r(P_4) + \frac{1}{2} r(P_5)$

Node 2 - *dangling*

- $r_0 = \left(\frac{1}{n}, \frac{1}{n}, \ldots\right)$

- $r_0 = \left(\frac{1}{n}, \frac{1}{n}, \ldots\right)$
- $r_0 \longrightarrow r_1$

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- $r_0 \longrightarrow r_1$
- $r_1 \longrightarrow r_2$

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- $r_0 \longrightarrow r_1$
- $r_1 \longrightarrow r_2$
- ...

- $r_0 = \left(\frac{1}{n}, \frac{1}{n}, \dots\right)$
- $r_0 \longrightarrow r_1$
- $r_1 \longrightarrow r_2$
- ...
- $r_k \longrightarrow r_{k+1}$

- $r_0 = \left(\frac{1}{n}, \frac{1}{n}, \ldots\right)$
- $r_0 \longrightarrow r_1$
- $r_1 \longrightarrow r_2$
- $\ldots$
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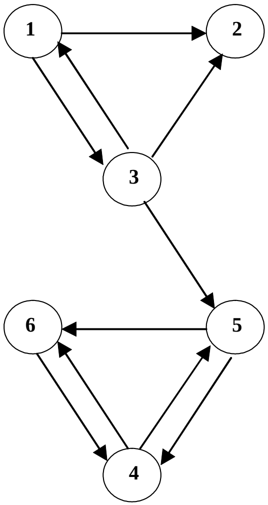
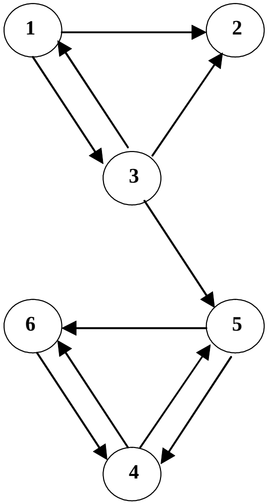
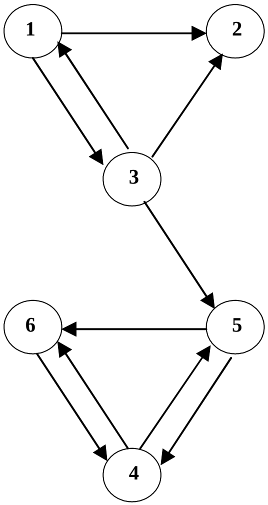
- $\ldots$



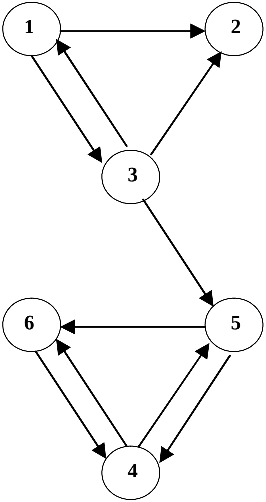
- $r_0 = \left(\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right)$



- $r_0 = \left(\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right)$
- $r_1\left(P_1\right) = \frac{1}{3} r_0\left(P_3\right) = \frac{1}{18}$



- $r_0 = \left(\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right)$
- $r_1(P_1) = \frac{1}{3} r_0(P_3) = \frac{1}{18}$
- $r_1(P_2) = \frac{1}{2} r_0(P_1) + \frac{1}{3} r_0(P_3) = \frac{5}{36}$



- $r_0 = \left(\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right)$
- $r_1(P_1) = \frac{1}{3} r_0(P_3) = \frac{1}{18}$
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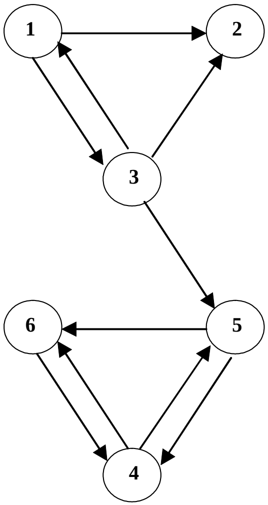
  $\frac{1}{2} r_0(P_1) + \frac{1}{3} r_0(P_3) = \frac{5}{36}$
- $r_1(P_3) = \frac{1}{2} r_0(P_1) = \frac{1}{12}$



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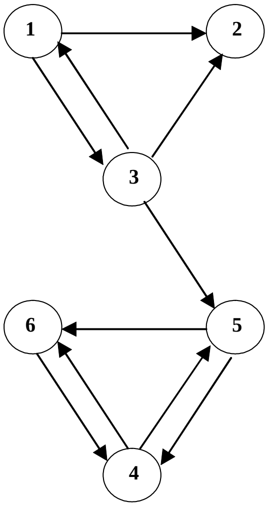
- $r_1\left(P_2\right) = \frac{1}{2} r_0\left(P_1\right) + \frac{1}{3} r_0\left(P_3\right) = \frac{5}{36}$
- $r_1\left(P_3\right) = \frac{1}{2} r_0\left(P_1\right) = \frac{1}{12}$
- $r_1\left(P_4\right) = \frac{1}{2} r_0\left(P_5\right) + r_0\left(P_6\right) = \frac{1}{4}$



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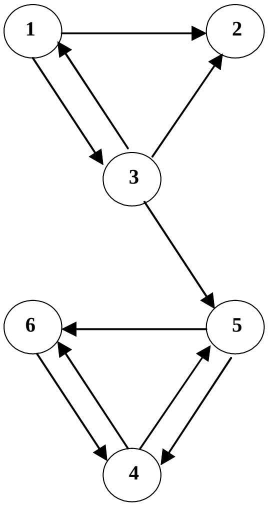
- $r_1\left(P_3\right) = \frac{1}{2} r_0\left(P_1\right) = \frac{1}{12}$
- $r_1\left(P_4\right) = \frac{1}{2} r_0\left(P_5\right) + r_0\left(P_6\right) = \frac{1}{4}$
- $r_1\left(P_5\right) = \frac{1}{3} r_0\left(P_3\right) + \frac{1}{2} r_0\left(P_4\right) = \frac{5}{36}$



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- $r_1(P_4) = \frac{1}{2} r_0(P_5) + r_0(P_6) = \frac{1}{4}$
- $r_1(P_5) = \frac{1}{3} r_0(P_3) + \frac{1}{2} r_0(P_4) = \frac{5}{36}$
- $r_1(P_6) = \frac{1}{2} r_0(P_4) + \frac{1}{2} r_0(P_5) = \frac{1}{6}$

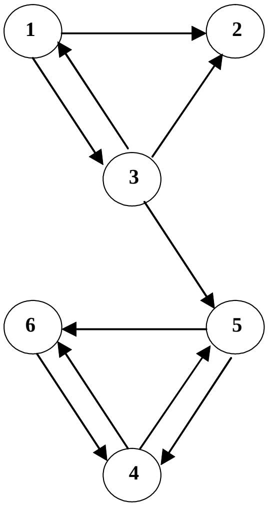
	r_0	r_1
P_1	$1/6$	$1/18$
P_2	$1/6$	$5/36$
P_3	$1/6$	$1/12$
P_4	$1/6$	$1/4$
P_5	$1/6$	$5/36$
P_6	$1/6$	$1/6$

	r_0	r_1	rank
P_1	$1/6$	$1/18$	6
P_2	$1/6$	$5/36$	$3-4$
P_3	$1/6$	$1/12$	5
P_4	$1/6$	$1/4$	1
P_5	$1/6$	$5/36$	$3-4$
P_6	$1/6$	$1/6$	2

	r_0	r_1	rank	r_2
P_1	$1/6$	$1/18$	6	$1/36$
P_2	$1/6$	$5/36$	$3-4$	$1/18$
P_3	$1/6$	$1/12$	5	$1/36$
P_4	$1/6$	$1/4$	1	$17/72$
P_5	$1/6$	$5/36$	$3-4$	$11/72$
P_6	$1/6$	$1/6$	2	$14/72$

	r_0	r_1	rank	r_2	rank
P_1	$1/6$	$1/18$	6	$1/36$	$5-6$
P_2	$1/6$	$5/36$	$3-4$	$1/18$	4
P_3	$1/6$	$1/12$	5	$1/36$	$5-6$
P_4	$1/6$	$1/4$	1	$17/72$	1
P_5	$1/6$	$5/36$	$3-4$	$11/72$	3
P_6	$1/6$	$1/6$	2	$14/72$	2

	r_0	r_1	rank	r_2	rank
P_1	$1/6$	$1/18$	6	$1/36$	$5-6$
P_2	$1/6$	$5/36$	$3-4$	$1/18$	4
P_3	$1/6$	$1/12$	5	$1/36$	$5-6$
P_4	$1/6$	$1/4$	1	$17/72$	1
P_5	$1/6$	$5/36$	$3-4$	$11/72$	3
P_6	$1/6$	$1/6$	2	$14/72$	2



Hyperlink matrix H :

	P_1	P_2	P_3	P_4	P_5	P_6
P_1	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0
P_2	0	0	0	0	0	0
P_3	$\frac{1}{3}$	$\frac{1}{3}$	0	0	$\frac{1}{3}$	0
P_4	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$
P_5	0	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$
P_6	0	0	0	1	0	0

Stochastic matrix: Every row is ≥ 0 and sums to 1.
H could be a stochastic matrix if it was not for the rows corresponding to the dangling nodes

- $$\underbrace{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6}\right)}_{r_0}\cdot\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$=\underbrace{\left(\frac{1}{18},\frac{5}{36},\frac{1}{12},\frac{1}{4},\frac{5}{36},\frac{1}{6}\right)}_{r_1}$$

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- Similarly:
$$r_2 = r_1 \cdot H = (r_0 \cdot H) \cdot H = r_0 \cdot H^2$$

- $$r_3 = r_2 \cdot H = ... = r_0 \cdot H^3$$

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- ... "Power Method"

QUESTIONS

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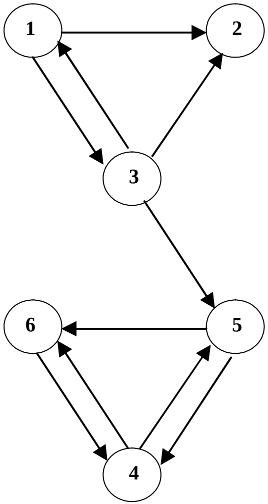
Page and Brin first had to deal with a number of problems:
Rank sink pages



-

	r_0	r_1	...	r_{13}
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P_6	1/6	1/6	...	1/5

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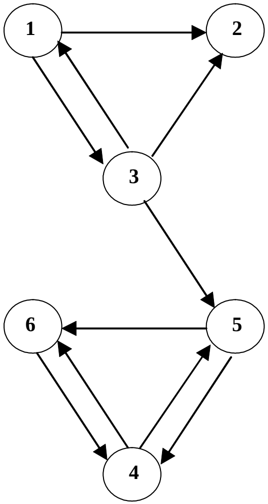

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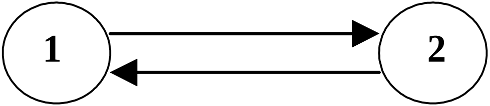
- Nodes 4,5,6 are a *link farm*.

- Another problem: *Cycles*



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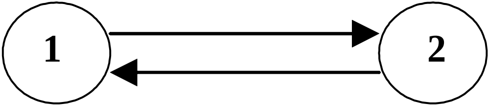

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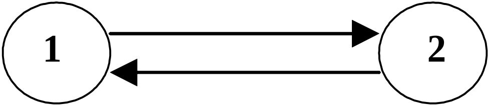

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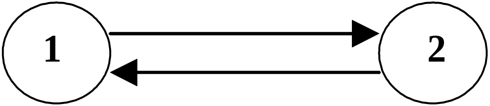

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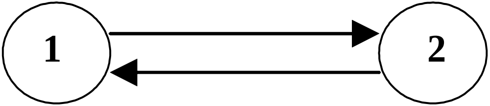

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- Flip-flop $\longrightarrow$ no convergence

Elements of the Markov chains theory:

- If H is a stochastic matrix and x_0 a stochastic vector then the sequence

$$\{x_0, x_1 = x_0 H, x_2 = x_1 H, x_3 = x_2 H, ...\}$$

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  is called a *Markov chain*.
- H is called the *transition probability matrix*.

Theorem (Markov, 1906)

If H is a **positive** *transition probability matrix of a Markov chain then this chain converges to a unique positive vector (called stationary vector) independently of the starting vector.*

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- Our example:

$$H \to S = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

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- S is stochastic! (But not yet positive)

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- and $\alpha \in (0,1)$ controls the proportion of the time RS follows the *hyperlinks* as opposed to *teleportation*

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- Hence any Markov chain with G is guaranteed to converge to a unique positive vector, independently of the starting vector.
- Actually used $\alpha \approx .85$

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- $G = 0.9S + 0.1E = \frac{1}{60}\begin{pmatrix} 1 & 28 & 28 & 1 & 1 & 1 \\ 10 & 10 & 10 & 10 & 10 & 10 \\ 19 & 19 & 1 & 1 & 19 & 1 \\ 1 & 1 & 1 & 1 & 28 & 28 \\ 1 & 1 & 1 & 28 & 1 & 28 \\ 1 & 1 & 1 & 55 & 1 & 1 \end{pmatrix}$

- Google's PageRank vector is the stationary vector of G which is

$$(.03721, .05396, .04151, .3751, .206, .2802)$$

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- Random surfer spends 3.721% of their time on P_1, etc...
- The importance ranking therefore is: $P_4, P_6, P_5, P_2, P_3, P_1$.

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- The stationary point of any Markov chain with transition matrix G is an eigenvector of G corresponding to the eigenvalue 1

Perron-Frobenius theory of Linear Algebra solves the eigenvector-eigenvalue problem for non-negative matrices.

Theorem (Perron, 1912)

If G is a positive, stochastic matrix then $\lambda = 1$ is an eigenvalue of G and G has a unique positive eigenvector (up to multiples).

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- Total: $\approx 50n$ operations (recall $n > 8$ billion)
- CONCLUSION: The power method with Google matrix is *very* fast!

THANK YOU

Next session in this room at 12.00:

"Careers, degrees and mathematics"