

# Positive Finite Volume Scheme for Laplace Equation on Arbitrary 2D Unstructured Meshes.

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## Abstract

A discrete Laplace operator is often considered as a good model for investigating a discretization of partial differential equations which contain second-order diffusion operator  $\nabla \cdot (D\nabla)$ . Solving these equations on unstructured grids, grid adaptation to the solution may lead to stretched cells with large (close to the value of  $\pi$ ) angles. A finite volume discretization of the diffusive terms has not been investigated sufficiently by now to overcome "bad" effects arising from geometric degeneration of stencils on arbitrary unstructured grids. One of the basic issues concerning this problem is that developed finite volume schemes are nonpositive in most cases, so one can expect strong oscillations or even divergency of solution.

We solve numerically the Laplace equation with Dirichlet boundary conditions in the unit square  $\bar{\Omega}$  by using a finite volume scheme, an arbitrary (i.e. with no restriction for triangle's angles) triangulation of  $\Omega$  being considered. For the scheme with a linear reconstruction, we state a control volume which guarantees a positive finite volume scheme on any unstructured grid. The control volume is defined by a property of the analytical solution to the equation and does not depend on grid geometry. For those problems where the choice of the control volume is prescribed a priori, we suggest how to correct the stencils to improve positivity of the scheme with linear reconstruction on arbitrary nonDelaunay triangulation. A hypothesis about generalization of a Delaunay criterion in the case of higher order reconstruction is formulated to predict a behaviour of the scheme coefficients.

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