MSM3P17 Computability – Example Sheet 2

All students will be asked to hand in solutions to the first 3 questions for assessment (not all of which will be marked). 4th year students will additionally be asked to hand in a solution to Question 4. You should place your solutions in my (=Deryk Osthus) pigeonhole by **1pm on Monday 14th February**. The second example class will take place on Tuesday, 8th February at **1pm** in LRC.

1. Suppose that you have a single tape Turing machine M' whose tape is only infinite in one direction, i.e. there is a cell where the TM cannot move to the left of this cell. Describe how one can simulate a standard single tape Turing machine M on M' in linear time. (Hint: consider the proof of Theorem 1.1. Does you need markers this time? What is the alphabet of M'? Does M' have more states than M?)

2. (a) A string w is an odd palindrome if it has odd length and if you obtain the same string when reversing the order of its symbols, e.g. if $w = 0110110$. Describe explicitly a single or multi-tape NTM which accepts exactly the set of odd palindromes w over the alphabet $\{0, 1\}$ in time at most $|w| + 1$ and explain in 2-3 sentences how and why it works. (Hint: Use 2 tapes.)

   (b) Write down all configurations (for each tape separately) for a computation ending in an accepting state and for a computation ending in a non-accepting state when the input is 010.

3. Describe briefly in words a 2-tape TM T which does the following. Initially, the 2 tapes contain a string y on tape 1 and x on tape 2, where $x, y \in \{0, 1\}^*$ and T should determine whether y contains x as a substring and accept precisely if it does (e.g. if $y = 010101$ it should accept if $x = 101$ but not if $x = 1001$).

   (a) T should be an NTM which works in time which is linear in the length $|y| + |x|$ of x and y.

   (b) T should be a DTM which works in ime which is quadratic in the length $|y| + |x|$ of x and y.

4. Describe informally an NTM which runs in linear time and accepts precisely those binary strings which contain some substring of length 20 at least twice (i.e. set of those strings of the form $axyxz$, with $|x| = 20$).