A Group of Order 8 That's Hard: Indecomposable String Modules for the Dihedral Groups

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Green's Indecomposability Criterion: If M is an indecomposable KH-module, $H \trianglelefteq\trianglelefteq G$, and $|G : H| = p^a$, then $M \uparrow^G$ is indecomposable.

Mackey Decomposition Theorem: If M is a KH_1-module, then

$$(M \uparrow^G) \downarrow_{H_2} = \bigoplus_{t \in T} (M \downarrow_{H_1^t \cap H_2})^t \uparrow^{H_2},$$

where T is a set of (H_1, H_2)-double coset representatives.

Mackey Tensor Product Theorem: If M and N are KH-modules, then

$$M \uparrow^G \otimes N \uparrow^G = \bigoplus_{t \in T} (M^t \downarrow_{H^t \cap H} \otimes N \downarrow_{H^t \cap H}) \uparrow^G,$$

where T is a set of (H, H)-double coset representatives.

Theorem (Conlon): Tensor products behave as in the table below.

$n \leqslant n'$	A_n	B_n
$A_{n'}$	$nn'D \oplus A_{n+n'}$	$n(n'+1)D \oplus A_{n'-n}$
$B_{n'}$	$n(n'+1)D \oplus B_{n'-n}$	$nn'D \oplus B_{n+n'}$
$C_{n'}(\pi')$	$nn'mD \oplus C_{n'}(\pi')$	$nn'mD \oplus C_{n'}(\pi')$
$C_{n'}(\infty)$	$nn'D \oplus C_{n'}(\infty)$	$nn'D \oplus C_{n'}(\infty)$
$n \leqslant n'$	$C_n(\pi)$	$C_n(\infty)$
$A_{n'}$	$nn'mD \oplus C_n(\pi)$	$nn'D \oplus C_n(\infty)$
$B_{n'}$	$nn'mD \oplus C_n(\pi)$	$nn'D \oplus C_n(\infty)$
$C_{n'}(\pi')$	X	$nn'mD$
$C_{n'}(\infty)$	$nn'mD$	$n(n'-1)D \oplus 2C_n(\infty)$

The remaining entry X is $nmn'm'D$ if $\pi \neq \pi'$, and $nm(n'm-1)D \oplus 2C_n(\pi)$ if $\pi = \pi'$.

Corollary: The modules $C_n(\pi)$, D and K are algebraic, and the modules A_n and B_n are not.