The Character Degrees of Symmetric Groups

David A. Craven

University of Oxford

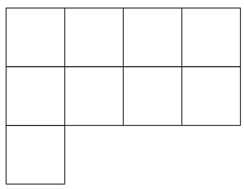
28th February, 2007

Partition of n: sequence of positive integers $(\lambda_1, \ldots, \lambda_r)$ such that

$$\sum_{i=1}^{r} \lambda_i = n.$$

The λ_i are the parts.

Partitions can be represented as tableaux, with number of boxes in each row equal to size of each part; e.g.,

$(4, 4, 1) \leftrightarrow$ 

Number of partitions of n is same as number of conjugacy classes of S_n.

There is a bijection between partitions of n and irreducible ordinary characters of S_n.

Write χ^λ for character corresponding to λ. The degree is given by the **Hook Order Formula**.

Let x be a box in λ. Then $h(x)$ equals the number of boxes below x, plus those to the right of x, plus 1. We have

$$\chi^\lambda(1) = \frac{n!}{\prod_{x \in \lambda} h(x)}$$

Conjugate partition λ^c of λ: partition whose tableau is reflection of λ.

For every λ, both λ and λ^c have the same hook numbers. Are there any others?

Cluster: set of partitions with same hook numbers.

λ is a partition, with r rows, c columns.

Write $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_r)$.

Let p be an integer with $1 \leqslant p \leqslant r-1$.

Remainder of λ: bottom $r-p$ rows.

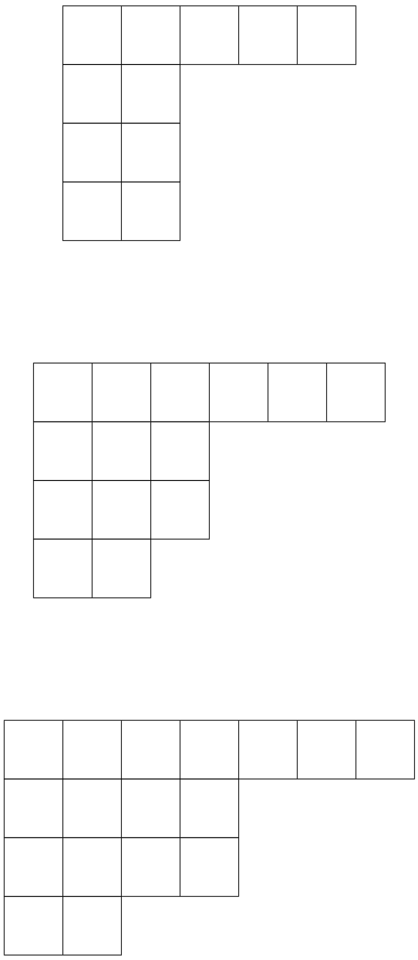
Rump of λ: top p rows.

Front section of λ: front piece of the rump.

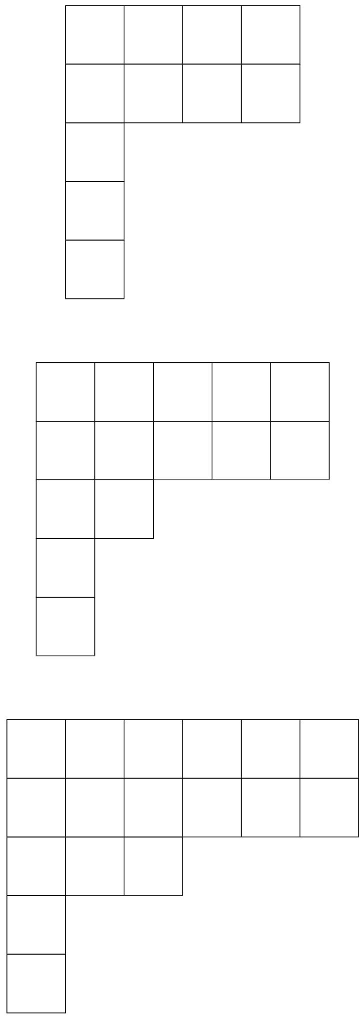
Extension of λ: the number $\lambda_p - \lambda_{p+1}$.

Periodic cluster: can add boxes to first p rows of all partitions and still get a cluster.

$(5+n, 5+n, 2+n, 2)$



$(4+n, 4+n, 1+n, 1, 1)$



$$E((5,5,3,2))$$