A Group of Order 604800

That's Easy: Octonians, $G_2(q)$

and J_2

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Basis for Algebra of Octonians

K-algebra of octonians is generated by $\{1, i_j\}$ over K with multiplication

	i_1	i_2	i_3	i_4	i_5	i_6	i_7
i_1	-1	i_4	i_7	$-i_2$	i_6	$-i_5$	$-i_3$
i_2	$-i_4$	-1	i_5	i_1	$-i_3$	i_7	$-i_6$
i_3	$-i_7$	$-i_5$	-1	i_6	i_2	$-i_4$	i_1
i_4	i_2	$-i_1$	$-i_6$	-1	i_7	i_3	$-i_5$
i_5	$-i_6$	i_3	$-i_2$	$-i_7$	-1	i_1	i_4
i_6	i_5	$-i_7$	i_4	$-i_3$	$-i_1$	-1	i_2
i_7	i_3	i_6	$-i_1$	i_5	$-i_4$	$-i_2$	-1

Doesn't work in characteristic 2!

$$2x_1 = ai_4 + i_6 + bi_7 \qquad 2x_8 = -ai_4 + i_6 - bi_7$$

$$2x_2 = ai_2 + bi_3 + i_5 \qquad 2x_7 = -ai_2 - bi_3 + i_5$$

$$2x_3 = -i_1 - bi_4 + ai_7 \qquad 2x_6 = -i_1 + bi_4 - ai_7$$

$$2x_4 = 1 - ai_3 + bi_2 \qquad 2x_5 = 1 + ai_3 - bi_2$$

With respect to this basis, we have the multiplication as given below.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
x_1	0	0	0	0	x_1	$-x_2$	x_3	$-x_4$
x_2	0	0	x_1	x_2	0	0	$-x_5$	$-x_6$
x_3	0	$-x_1$	0	x_3	0	$-x_5$	0	x_7
x_4	x_1	0	0	x_4	0	x_6	x_7	0
x_5	0	x_2	x_3	0	x_5	0	0	x_8
x_6	x_2	0	$-x_4$	0	x_6	0	$-x_8$	0
x_7	$-x_3$	$-x_4$	0	0	x_7	x_8	0	0
x_8	$-x_5$	x_6	$-x_7$	x_8	0	0	0	0