Algebraic Modules for Finite Groups

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August, 2007

Algebraic Modules

Algebraic module: One that satisfies a polynomial in $\oplus$ and $\otimes$. (Alperin, 1976)

Examples: Projective modules, permutation modules, simple modules for p-soluble groups (Feit, 1979).

Behaves well under sums, summands, tensor products, induction, restriction, source-taking, Green correspondence.

Conjecture: If $\hat{M}$ is an extension of M from a normal subgroup, then $\hat{M}$ is algebraic if M is.

Periodicity and Algebraicity

Theorem: Suppose that M is periodic and algebraic. Then $\Omega^i(M)$ is algebraic for all i.

Theorem: Suppose that M is non-periodic and algebraic. Then for all $i \neq 0$, $\Omega^i(M)$ is non-algebraic.

In some sense, 'most' non-periodic modules are non-algebraic. Can we prove more?

Auslander–Reiten Quiver

M an indecomposable module, complexity 3.

$M \in \Gamma \subseteq \Gamma_s(KG)$, a component of the AR-quiver.

Γ is a wild component, so of type A_∞.

Theorem: There is at most one algebraic module on Γ, and it lies at the end.

This can be extended to a wider class of module; e.g., Sylow p-subgroups not C_p or $C_p \times C_p$, and $p \nmid \dim M$. What can be said for $C_p \times C_p$?

$C_3 \times C_3$-modules

M an absolutely indecomposable module.

$\mathbf{3 \nmid \dim M}$: no nice description, even conjecturally.

$\mathbf{3 \mid \dim M}$: conjecturally a nice description.

Conjecture: M algebraic iff M periodic.

Evidence: true for GF(3) and $\dim M = 3, 6$. There are 239 such modules.

This should extend to $C_p \times C_p$.