Alternating subgroups of exceptional groups of Lie type, Errata

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Note that none of the theorems in the paper is false, all errors below are missing words, etc., from the proofs, except for one case where the proof needs to be repaired. Indeed, since publication of this paper, results have improved, and at the moment the exceptions to Theorem 3 with G of type F_4 can be removed (the first of these appears in the author's Memoir of the AMS 'Rank 1 subgroups of exceptional algebraic groups, the second is due to Andrea Pachera and is not yet published). At the time of writing, the E_6 and ${}^2E_6$ cases from Theorem 3(iv) have not been completely removed, although it has been proved that such a subgroup H is Lie imprimitive. (Thus H lies in a member of $\mathscr{X}$, but not necessarily one from $\mathscr{X}^\sigma$. A sketch of such a proof has been written, but it has not been written up.) Again, this is due to Andrea Pachera in as-yet unpublished work.

Note that Theorems 2 to 5 only dealt with the simple group. For almost simple groups (and novelty maximal subgroups), the reader is directed towards the work on F_4 and E_6 currently in preparation, or the appendix in the updated version of this work, which outlines how to 'upgrade' these theorems.

I distinguish four types of corrections, in order of increasing seriousness:

(Extra) Additional information that was not available at the time of writing, or that I did not know about.

(Improve) Typographical issues, where what is written is still correct, but there is a nicer way of phrasing it, or I could choose a better symbol.

(Typo) Typographical errors, where I have spelled a word wrongly, used the wrong symbol, and so on.

(Error) Errors in proofs or statements.

When I give each correction, I will label it with one of these monikers.

(Typo) p460, Defn 1.7, missing a dim in pressure formula.

(Improve) p461, Prop 1.9. This is a bit clunky as written. A better version is as follows:

Let $G = O^p(G)$ be a finite group and let M be a kG-module. Suppose that, for every composition factor V of M, $H^1(G,V) = J^1(G,V^*)$. Suppose that M has at least one trivial composition factor.

If M has pressure n, then no subquotient of M has pressure greater than n or less than $-n$. In particular, if M has non-positive pressure then M has a trivial submodule or quotient. Furthermore, if M possesses a composition factor V such that the dimension of $H^1(G,V)$ is greater than the pressure of M, then M possesses a trivial submodule or quotient.

(Error) p461, just after the proof of 1.9, should read 'Modules of pressure 1 **with no trivial submodules or quotients**'.

(Typo) p462, l-13, the sentence should read 'If $H = \mathrm{Sp}_6(2)$, then again there is **no** copy'.

(Typo) p465, Lemma 2.1, line 1, should be a full stop rather than a comma at the end of the first sentence.

(Error) p467, Lemma 3.2, end of proof. Should read '...three 2_i composition factors for **every two trivial factors**'.

(Improve) p468, Proposition 4.1. Note that this proposition relies in Theorem 4: we assume that H is in a positive-dimensional subgroup to begin with. Theorem 4 does not require 4.1, so this is allowed.

(Typo) p474, I don't know how this happened, but something went wrong with the references to Frey's paper [7] in the proof of 5.1. The references should instead be to Frey's Memoir of the AMS article 'Conjugacy of Alt_5 and $\mathrm{SL}(2,5)$ subgroups of $\mathrm{E}_8(\mathbb{C})$'.

(Extra) p478, with regards characteristic 0. For F_4 this set of composition factors was proved to not yield a new maximal subgroup by Andrea Pachera, but his results are not yet published.

For E_6, he has shown that the set of factors for $\mathrm{Alt}(6)$ lies in a member of $\mathscr{X}$, but not (at the time of writing) in a member of $\mathscr{X}^\sigma$.

For the subgroup $3 \cdot \mathrm{Alt}(6)$, this does not occur. This was mentioned in [8], but that relies on a paper of Cohen and Wales. This paper does not prove some of its claims, and indeed Theorem 7.1 from that paper is false (as there are missed examples). I prove this explicitly in work on the maximal subgroups of F_4 and E_6 currently in preparation.

(Typo) p479, l7, $y = (1,2,3,4,5)$ should be $u = (1,2,3,4,5)$.

(Improve) p479, l-12, added brackets help readability:

$$10_2^{\oplus 3} \oplus 10_3 \oplus (4_1/4_2) \oplus (4_2/4_1).$$

(Extra) p480, third paragraph. The case F_4 has been completed in the author's later paper, as promised in this paragraph. See the start of this document for details.

(Typo) p480, Prop. 6.2(iii), should read 'σ-stable 2-space of V_{min}'.

(Error) p482, Prop. 6.3(ii), should read H fixes a line **or hyperplane** on $V_{\min}$.

(Error) p483, top. This is incorrect: all four sets of composition factors have corresponding sets on $L(G)$, one of which could in theory not stabilize a line on $L(G)$. However, one may show, for example in Proposition 10.3 of the author's work on medium-rank subgroups, that only the first and third cases can occur, and both of these yields stabilized lines on $L(G)$, so the statement is still true. This is also performed in the updated version of this paper.

(Typo) p483, l3, the second set of factors should be $8_1, 4_1^7, 4_2^7, 1^{14}$.

(Improve) p486, Prop 7.3(v) and (vi), readability can be improved by adding brackets around the summands, so $(4/14) \oplus (14/4^*)$ and $(8/6) \oplus (6/8)$.

(Typo) p492, the third line of Step 1. The fourth set of factors in the displayed equation is wrong, and the third is slightly wrong; they should be

$$(20, 20^*)^2, 14^4, 6^8, (4, 4^*)^7, 1^8, \quad 64^2, 20, 20^*, 14^4, 6^2, 1^4.$$

(The fourth set is given correctly when that case is considered later in the proof.) The third is also given incorrectly as 20^4 rather than $(20, 20^*)^2$ in l-3 from that page.

(Improve) p493, the second displayed equation on that page, put brackets around two summands for clarity:

$$1 \oplus (1, 1/14, 20, 20/1, 1, 1).$$

(Typo) p493, l-8, 10 should be 20 and $6^*/4/^*$ should be $6^*/4/6^*$.

(Error) p494, l10, we should add $6^3, 20^2$ to the list of possible socles. We also add $6^3, 20^2$ to the next sentence as well, so it should be 'However, with $6^3, 20$ and $6^3, 20^2$' as well.

(Error) p496, at the top, note that the socle cannot be either 7 or 21 as the appropriate radicals are $21/7$ and $21/7, 21/21$ respectively.

(Improve) p496, l-10, the statement $\mathrm{Ext}^1(35, 1, 21/35)$ needs brackets to make more sense. Then it becomes $\mathrm{Ext}^1(35, (1, 21/35))$.

(Typo) p498, l22, the displayed equation for filtration of A_7 on $L(G)$, the third radical layer should be $L(\lambda_5)$, not $L(\lambda_3)$.

(Typo) p499, l-18, the convention for extensions of modules is an extension of A by B has B as the submodule. So in this case it should be an extension of $\mathrm{soc}(M/N)$ by N.