INDUCING FIXED POINTS IN THE STONE-ČECH COMPACTIFICATION

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Abstract. If f is an autohomeomorphism of some space X, then βf denotes its Stone-Čech extension to βX. For each $n \leq \omega$, we give an example of a first countable, strongly zero-dimensional, subparacompact X and a map f such that every point of X has an orbit of size n under f and βf has a fixed point. We give an example of a normal, zero-dimensional X such that f is fixed-point-free but βf is not. We note that it is impossible for every point of X to have an orbit of size 3 and βX to have a point with orbit of size 2.

For every Tychonoff space X there is a unique compact, Hausdorff space βX, the Stone-Čech compactification of X, which contains X as a dense subspace and has the property that every autohomeomorphism and every continuous $\mathbb{R}$-valued map f on X can be uniquely extended to one on βX, denoted βf (see, for example, [E]). Even if an autohomeomorphism f has no fixed points, βf may do. Such induced fixed points can be regarded as ideal and, following work by van Douwen and Watson, we describe examples of spaces with fixed-point-free autohomeomorphisms which nevertheless have ideal fixed points. For each $n \leq \omega$, we give an example of a first countable, strongly zero-dimensional, subparacompact X and a map f such that every point of X has an orbit of size n under f and βf has a fixed point. Since neither these examples, nor those described by Watson in [W], are normal, we also give an example of a normal, zero-dimensional X such that f is fixed-point-free but βf is not. This example is based on the space described in [D]. Answering a question from [W], we note that it is impossible for every point of X to have an orbit of size 3 and βX to have a point with orbit of size 2. We also show that a set can be topologized so that a fixed-point-free permutation is an autohomeomorphism with an ideal fixed point if and only if the set is uncountable.

1. Preliminaries.

We are interested here in autohomeomorphisms of Tychonoff spaces and their Stone-Čech extensions so all spaces are Tychonoff.

We use Greek (π) to denote a permutation on a set and Roman (f) when the set is topologized and the permutation is an autohomeomorphism. If A is a subset of some set X, we denote the image of A under a map π by $\pi``A$. As usual, we regard natural numbers as ordinals and an ordinal as the set of all smaller ordinals. The set of all natural numbers is

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written ω and the set of all countable ordinals is written ω_1. If j divides n then we write $j \mid n$.

For any permutation π of X and x in X, we denote the orbit $\{\pi^n(x) : n \in \mathbb{Z}\}$ of x by $\mathrm{orb}_\pi(x)$ or simply $\mathrm{orb}(x)$ if π is clear. If $|\mathrm{orb}(x)| = n$ for any $n \le \omega$, then we say that x has order n and that $\mathrm{orb}(x)$ is an n-cycle. We can assign a sequence of cardinals $\sigma(\pi)$ to π, describing its cycle structure: $\sigma(\pi) = (\kappa_n)_{n\in\omega}$, where κ_0 is the number of ω-cycles and, if $0 < n$, κ_n the number of n-cycles. We call π a rotation of period n, if all entries in the sequence $\sigma(\pi)$ are zero except κ_n. Watson [W] calls a rotation of period 2 a reflection, and a rotation of period ω a translation. If π is a permutation of a set X and P is some topological property (or space), then we say that π is *P-realizable* if there is a topology on X having the property P (or so that X is homeomorphic to P) with respect to which π is an autohomeomorphism of X.

The Stone-Čech extension of a map f is written βf. In keeping with Watson [W], we shall say that a Tychonoff space X is FAE (fixed-point-free autohomeomorphisms extend) if βf is fixed-point-free whenever f is fixed-point-free autohomeomorphism of X.

Our basic set-theoretic reference is [K]. Undefined topological terms may be found in [E].

2. Autohomeomorphic rotations with ideal fixed points.

In [BK], Blaszczyk and Kim prove that every strongly zero-dimensional, paracompact space is FAE and, in [D], van Douwen proves that every paracompact space of finite Lebesgue covering dimension (dim) is FAE, and gives an example of a locally compact, separable, metric (hence paracompact) space which is not FAE (see Section 3). This leaves open the general situation in finite dimensional, particularly (strongly) zero-dimensional, spaces. (Two sets are completely separated if there is a continuous $f : X \to [0,1]$ such that $f``A = \{0\}$ and $f``B = \{1\}$. A space is zero-dimensional if it has a neighbourhood base at each point consisting of clopen sets, and strongly zero-dimensional if for every two completely separated sets A and B there is a clopen set U containing A but disjoint from B.) Watson [W] has described two first countable, zero-dimensional spaces, one with a reflection witnessing that it is not FAE, the other with a translation. In 2.3 we describe first countable, strongly zero-dimensional, subparacompact (Moore) examples with rotational autohomeomorphisms of arbitrary order witnessing non-FAE. These spaces have cardinality ω_1, which allows us to completely determine when a permutation of a set X can be (Tychonoff, not FAE)-realized.

A point of βX can be seen as an ultrafilter of functionally closed sets of X. If f is a fixed-point-free autohomeomorphism of X and p is some point of βX fixed by βf, then it is easy to see that f does not fix every subset of X in p. One way, therefore, to find a non-FAE is to restrict the (ultrafilters of) functionally closed sets. This idea is used in [BK] where the space $X = \{-1,0,1\}^{\omega_1} - \vec{0}$ is shown to be non-FAE (the map "$x \mapsto -x$g" is a witness to this, there being only one point in $\beta X - X$). Watson also uses this idea to construct another space with a reflective autohomeomorphism witnessing non-FAE, basing it on an example due to van Douwen, having two disjoint closed copies of ω_1, which are not separated by disjoint open sets. Here we construct our examples from a base space Z, which has two disjoint closed (discrete) subsets which can not be functionally separated. Again, Z relies on the properties of ω_1. Z was used to different effect in [GT].

Without the requirement of first countablity, the problem is easy:

Two simple examples 2.1: Let $(\omega_1+1)^2$ have the usual product topology. Using the pressing down lemma, it is easy to show (and is indeed well known) that $\omega_1 \times \{\omega_1\}$ and the diagonal can not be separated by disjoint open sets in the subspace $W = (\omega+1)^2 - \{(\omega_1,\omega_1)\}$. Let M' be the space $W \times 2$ and let f' be the autohomeomorphism taking the point (α,β,i) of M' to (β,α,j), $i \ne j \in 2$. Let q be the quotient map identifying (α,ω_1,i) with (ω_1,α,j), let M be the quotient space $q``M'$, and let $f = q \circ f'$. The map f is an autohomeomorphic reflection.

Let N have point set $\omega_1 \times (\omega_1 + 1) \times 2$. Let points of $\omega_1 \times \omega_1 \times 2$ be isolated. If $(\beta, \alpha]$ is a neighbourhood of $\alpha \in \omega_1$ with its usual topology, then let $A_{\delta,\beta}(\alpha, \uparrow)$ be the subset $\bigcup_{\delta<\gamma<\alpha}((\beta, \alpha] \times \{\gamma\})$ of ω_1^2 where $\delta < \alpha$, and let $A_{\delta,\beta}(\alpha, \downarrow)$ be the subset $\bigcup_{\delta<\gamma}((\beta, \alpha] \times \{\gamma\})$ where $\delta > \alpha$. For $\beta, \delta < \alpha < \gamma$ and $i \neq j \in 2$, let $B_{\gamma,\delta,\beta}(\alpha, i)$ be the set $((\beta, \alpha] \times \{\omega_1\} \times \{i\}) \cup (A_{\delta,\beta}(\alpha, \uparrow) \times \{i\}) \cup (A_{\gamma,\beta}(\alpha, \downarrow) \times \{j\})$. Let $\mathcal{T}$ be the topology on N generated by the collections $\{\{\alpha, \beta\} : \alpha, \beta \in \omega_1\}$ and $\{B_{\gamma,\delta,\beta}(\alpha, i) : \beta, \delta < \alpha,\ i \in 2\}$. By the pressing down lemma, $(\alpha, \omega_1) \times \{\omega_1\} \times \{0\}$ and $(\beta, \omega_1) \times \{\omega_1\} \times \{1\}$ can not be separated by disjoint open sets for any α and β in ω_1. The map $f : (\alpha, \beta, i) \mapsto (\alpha, \beta, j)$, where $\alpha < \omega_i$, $\beta \leq \omega_1$, and $i \neq j \in 2$, defines an autohomeomorphic reflection of N.

In both cases βf has a fixed point: for example, in M if p is in the closure of $\omega_1 \times \omega_1 \times \{0\}$ and $\beta f p$ is distinct from p, then disjoint open neighbourhoods of p and $\beta f p$ in βM trace to disjoint open neighbourhoods of (most of) $\omega_1 \times \omega_1 \times \{0\}$ and $\omega_1 \times \omega_1 \times \{1\}$. $\square$

The space Z 2.2: Let $W = \omega_1 \times (\omega + 1)$. Let $W_0 = \omega_1 \times \omega$ and $W_1 = \omega_1 \times \{\omega\}$. For each $\gamma \in \omega_1$, let $W(\gamma) = (\gamma, \omega_1) \times (\omega + 1)$ and $W_1(\gamma) = W_1 \cap W(\gamma)$. For each α in ω_1, let $\{\alpha_n\}_{n\in\omega}$ be an increasing sequence cofinal in α (if α is a successor then let each α_n be $\alpha - 1$). Let each point of W_0 be isolated and let a basic open neighbourhood of a point (α, ω) in W_1 take the form $B_n(\alpha) = \{(\alpha, \omega)\} \cup \bigcup_{n\leq m}(\alpha_m, \alpha] \times \{m\}$.

With this topology W is the basic Reed space over ω_1 (see [R]) and is therefore a zero-dimensional Moore space. The following claim is proved in [GT]. It is a simple modification of the proof that every real-valued continuous function on ω_1 is eventually constant.

Claim 1. *Every continuous $\mathbb{R}$-valued function on W is eventually constant on W_1, that is, for any $\mathbb{R}$-valued map f on W, there is some $\gamma \in \omega_1$ such that f is constant on $W_1(\gamma)$.*

Now let $\mathbb{Z}^* = \mathbb{Z} \cup \{-\infty, \infty\}$ be the two point compactification of the integers (so that $\mathbb{Z}^*$ is homeomorphic to the subset $\{\pm 1\} \cup \{\pm 1 \mp 1/n : 0 < n\}$ of $\mathbb{R}$). Let $Z'' = W \times \mathbb{Z}^*$ have the usual product topology, and $Z' = Z'' - (W_0 \times \{\pm\infty\})$. Partition W_1 into disjoint sets S_0 and S_1 such that $\{\alpha : (\alpha, \omega) \in S_i\}$ is stationary for each $i \in 2$. Let Z be the quotient space formed by identifying the point $(a, 2n)$ of Z' in $S_0 \times \{2n\}$ with the point $(a, 2n + 1)$ of $S_0 \times \{2n + 1\}$, and the point $(a, 2n + 1)$ of $S_1 \times \{2n + 1\}$ with the point $(a, 2n + 2)$ of $S_1 \times \{2n + 2\}$, for every n in $\mathbb{Z}$. Let q denote the quotient mapping defined by this identification. It is not too hard to show that Z is a zero-dimensional (Moore) space.

Let W_1^- and W_1^+ denote the closed, discrete subsets $W_1 \times \{-\infty\}$ and $W_1 \times \{\infty\}$ of Z.

Claim 2. *For any $\mathbb{R}$-valued function $f : Z \to [0, 1]$ there is some $\gamma \in \omega_1$ for which f is constant on $q``(W_1(\gamma) \times \mathbb{Z}^*)$. Hence $W_1^-(\gamma)$ and $W_1^+(\gamma)$ are disjoint closed sets which cannot be functionally separated.*

Proof of Claim 2. Let $f : Z \to \mathbb{R}$ be any continuous map. For any n in $\mathbb{Z}$, $f \restriction_{W\times\{n\}}$: $W \times \{n\} \to \mathbb{R}$ is continuous, and by Claim 1 there is some $\gamma_n \in \omega_1$ such that $f \restriction_{q``(W\times\{n\})}$ is constant on $q``(W_1(\gamma_n) \times \{n\})$. Let $\gamma = \sup \gamma_n$. Because of the identification of points in Z, f is constant on $W_1(\gamma) \times \mathbb{Z}$. Hence there is some $\gamma' \geq \gamma$ such that f is constant on $(W_1(\gamma') \times \{-\infty\}) \cup (W_1(\gamma') \times \{\infty\})$, proving the claim.

Since Z is a Moore space, it is subparacompact. (A space is subparacompact if every open cover has a refinement of closed sets that is a countable union of locally finite collections. In the class of collectionwise normal spaces subparacompactness coincides with paracompactness.) To see that it is strongly zero-dimensional, let C and D be functionally separated closed sets. By Claim 2, it is not possible for both C and D to have uncountable intersection with $W_1 \times \mathbb{Z}^*$, so there is some successor α for which, say, C does not meet $W_1(\alpha) \times \mathbb{Z}^*$. Since α is a successor, $Z(\alpha) = W(\alpha) \times \mathbb{Z}^*$ is a clopen subset of Z. The clopen subset $Z - Z(\alpha)$ of Z is a countable, zero-dimensional space and is therefore strongly zero-dimensional. Moreover, $C \cap Z(\alpha)$ is clopen since C is closed and every point of this set is isolated in Z. Hence C and D can be separated by disjoint clopen subsets.

(It is also possible to give Z a coarser strongly zero-dimensional topology such that W_1^+ and W_1^- are homeomorphic to ω_1 and cannot be functionally separated.) $\Box$

Theorem 2.3. *For every $0 < n \leq \omega$, there is a first countable, strongly zero-dimensional, subparacompact space X and a rotational autohomeomorphism f of X of order n which has an ideal fixed point.*

Proof. The result is trivial if $n = 1$, so suppose that $1 < n \leq \omega$. If n is finite then take all integers $\mod n$. For each $j \in n$, let $Z(j)$ be a (distinct) copy of Z, let id_j be the identity map from $Z(j)$ to $Z(j+1)$, and for any subset A of Z let $A(j)$ denote the corresponding subset of $Z(j)$. Let X be the space formed by identifying points of $W_1^+(j)$ with the corresponding points of $W_1^-(j+1)$ (i.e. identify $(\alpha, \omega, +\infty)$ and $(\alpha, \omega, -\infty)$), and let W_j denote the resulting set. Let $W_j(\alpha)$ denote the subset of W_j corresponding to $W_1^+(\alpha)(j)$. The maps id_j generate an obvious autohomeomorphism f' of $\bigcup_{j \in n} Z(j)$ and, if f is the autohomeomorphism of X generated by f', then $|\operatorname{orb}(x)| = n$ for each x in X.

From Claim 2 it follows that, for any $j, k < n$, W_j and W_{j+1}, and hence W_j and W_k, are not functionally separated in X. Let C_α be the set $\overline{W_k(\alpha)}^{\beta X}$. Since $\{C_\alpha : \alpha \in \omega_1\}$ has the finite intersection property and βX is compact, there is some point p in $\bigcap_{\omega_1} C_\alpha$. Suppose that $\beta f(p) \neq p$. Then p and $\beta f(p)$ are functionally separated by some $\mathbb{R}$-valued function h. By the definition of f, $\beta f(p)$ is in $\overline{W_{k+1}}^{\beta X} - W_{k+1}$. Since $h \restriction_{Z(k+1)}$ is eventually constant, it functionally separates (unbounded subsets of) $W_1^-(\gamma)(k+1)$ and $W_1^+(\gamma)(k+1)$ for some γ, contradicting Claim 2. $\Box$

These spaces are not countably paracompact since Z is not (see [GT]). The space described in 3 is countably paracompact and normal, but not subparacompact (since it contains a copy of ω_1) and not *strongly* zero-dimensional.

Corollary 2.4. *A fixed-point-free permutation π of the set X can be (Tychonoff, not FAE)-realized if and only if X is uncountable.*

Proof. If X is countable then any Tychonoff topology on X will be strongly zero-dimensional and paracompact, and hence FAE, by the result of van Douwen's mentioned above. If X is uncountable then $X_n = \{x \in X : x \text{ has order } n\}$ is uncountable for some n. Let Y be a subset of X_n consisting of ω_1 many complete cycles. Topologize Y so that it is homeomorphic to the space of Proposition 2.2 and let every point of $X - Y'$ be isolated. $\Box$

3. A normal, zero-dimensional space that is not FAE.

Neither the examples in [BK], [W] nor those of Section 2 are normal, so here we describe a normal (in fact collectionwise normal), zero-dimensional space that is not FAE. We combine van Douwen's example of a locally compact, separable, metrizable, non-FAE space and Dowker's construction of a normal, zero-dimensional space which fails to be strongly zero-dimensional (see 6.2.20 [E])—so the space stands no chance of being strongly zero-dimensional (in fact it is zero-dimensional but has infinte Lebesgue covering dimension).

For convenience we outline van Douwen's description from [D]: Let S^n be the n-sphere and $a_n : S^n \to S^n$ be the antipodal map "$x \mapsto -x$". If $\mathcal{F}$ is a closed cover of S^n such that F and a_n"F are disjoint, then $\mathcal{F}$ has at least $n+2$ elements. Let M be the disjoint topological sum $\bigoplus_{0<n\in\omega} S^n$ and let $h : M \to M$ be the autohomeomorphism such that $h \restriction_{S^n} = a_n$. Suppose that βM has no fixed points. Since βM is compact, it has a finite cover $\mathcal{F}$ of closed sets F such that F and h“ F are disjoint. Let $\mathcal{F}_n = \mathcal{F} \cap S^n$. If n is large enough, then $|\mathcal{F}_n| \leq |\mathcal{F}| < n+2$, which is impossible.

Now, for each $n \in \omega$, let $\{S^n_\alpha : \alpha \in \omega_1\}$ be an increasing sequence of subsets of S^n such that

(1) if $\beta < \alpha$, then S^n_β is a subset of S^n_α;
(2) S^n_α is zero-dimensional subspace of S^n;
(3) a_n“ $S^n_\alpha = S^n_\alpha$, and;
(4) $\bigcup_{\alpha\in\omega_1} S^n_\alpha = S^n$.

Let $T_n = \bigcup_{\alpha\in\omega_1} S^n_\alpha \times \{\alpha\}$ and $T^*_n = T_n \cup (S^n \times \{\omega_1\})$ be subspaces of the Tychonoff product $S_n \times (\omega_1 + 1)$.

The proof that T_n is first countable, normal, zero-dimensional, but not strongly zero-dimensional, that βT_n is βT^*_n and that, in particular, $S^n \times \{\omega_1\}$ is a subset of βT_n is, almost verbatim, contained in Engelking's description of Dowker's space [E, 6.2.20].

Let $\mathbb{T} = \bigcup_{n\in\omega} T_n$. $\mathbb{T}$ is zero-dimensional, first countable, and normal. Since there is a copy of S^n in βT_n and βT_n is a subspace of $\beta\mathbb{T}$, $\beta\mathbb{T}$ contains a copy of M. Moreover, by 3), the fixed-point-free autohomeomorphism h of M induces a fixed-point-free autohomeomorphism χ of $\mathbb{T}$ such that the restriction of $\beta\chi$ to M is h. Since βM is a subspace of $\beta\mathbb{T}$ and the restriction of $\beta\chi$ to βM is βh, $\beta\chi$ has a fixed point.

Questions Notice that χ is a reflection and also that van Douwen's example shows that FAE is not preserved by infinite topological sums. One can ask whether it is preserved by countable (or arbitrary) products or by open maps. Presumably it is not. It is not peserved by perfect maps since the examples of 2.3 are perfect iamges of Z''. One might also ask whether van Douwen's result holds for finite dimensional, monotonically normal or GO spaces, for completely metrizable spaces, or for paracompact spaces with finite small inductive dimension. □

4. Ideal rotations.

Extending the notion of an ideal fixed point, one might define an ideal n-cycle of a map f to be an n-cycle of the map βf. In [W], Watson asks whether a rotational autohomeomorphism of order 3 can have an ideal 2-cycle, and for which sequences $\sigma(\pi)$ is π P-realizable when P is regular, compact, or metrizable, or $\mathbb{R}$, $\mathbb{Q}$, or $\mathbb{P}$ (the irrarionals). We close by pointing out that the first question has a negative answer. (Along with other results, we shall answer the second in a forthcoming paper, using a strengthening of 4.1 for zero-dimensional, compact scattered spaces.) Watson also asks for which $\sigma(\pi)$ and $\sigma(\phi)$ is π (Tychonoff)-realizable with ϕ $\beta\pi$-realizable. This question seems harder—for a start we do not know of any results, other than obvious restrictions, relating $|X|$ to $|\beta X|$.

Theorem 4.1. *Let f be an autohomeomorphism of the Hausdorff space X and let $0 < k$. If x has finite order $n \in \omega$ (has order ω) then for any $k \in \omega$ there is an open neighbourhood U of x such that every y in U has order greater than k or divisible by n (has order greater than k).*

For any finite collection of autohomeomorphisms $\{f_1, \ldots, f_n\}$ and $0 < k \in \omega$ there is an open neighbourhood U of x such that $|\operatorname{orb}_{f_j} x|$ divides $|\operatorname{orb}_{f_j} y|$ whenever $\operatorname{orb}_{f_j} x$ is finite, y is in U and $|\operatorname{orb}_{f_j} y| < k$

Proof. Let nr_k be the least multiple of n greater than k. Since X is Hausdorff, one can find an open neighbourhood W of x such that f^i“ $W \cap f^j$“ W is empty whenever for $0 \leq i, j \leq nr_k$ and $i \neq j \mod n$). Let U_k be the set $\bigcap_{0\leq r\leq r_k} f^{nr}$“ W. The result for points of order ω follows similarly and the result for finite collections of autohomeomorphisms follows immediately. $\square$

It is clear from 4.1 that no rotational autohomeomorphism of order 3 has an ideal reflected point in βX. In fact:

Corollary 4.2. *There is a Tychonoff space $X_{n,m}$ and a rotational autohomeomorphism $f_{n,m}$ of order m which has an ideal n-cycle if and only if $n \mid m$ or $m = n = \omega$.*

Proof. Necessecity is immediate by the lemma. For sufficiency, first suppose $n = m = \omega$ and let $X_{n,m}$ be $\mathbb{Z}$ and f be the shift, $f(x) = x+1$. By van Douwen's result, $X_{n,m}$ is FAE and we are done. Now suppose that n is finite and $m = nr$ for some $r \leq \omega$. Let X be any Tychonoff space with a rotational autohomeomorphism f of order r with an ideal fixed point p. For each $j < n$ let X_j be a distinct copy of X, p_j correspond to p, f_j be the corresponding selfmap, and let id_j be the identity map from X_j to X_{j+1}. Let $X_{n,m}$ be the disjoint topological sum of the X_j and, for x in X_j define $f_{n,m}(x) = f_{j+1}(\mathrm{id}_j(x)) = \mathrm{id}_j(f_j(x))$ (where j and $j+1$ are taken mod n). $\square$

If X is a P-space, i.e. countable intersections of open sets are open, then about each x of finite order n there is an open neighbourhood U such that every point of U has either infinite order or order divisible by n. The same is also true for zero-dimensional, compact, scattered spaces but it is not true in general.

For each $r > 2$, let $\{x_{r,i} : i < 2r+1\}$ be $2r+1$ distinct points. Let $f(x_{r,i}) = x_{r,i+1}$. $X = \{w_0, w_1\} \cup \{x_{r,i} : i < 2r+1, r \in \omega\}$ and define $f(w_i) = w_j$. Let each $x_{r,i}$ be isolated and topologize X so that w_0 is in the closure of $\{x_{r,i} : i < 2r \text{ is even}\}$, w_1 is in the closure of $\{x_{r,i} : i \text{ is odd}\}$ and f is an autohomeorphism.

Let Y be $\mathbb{Z} \times (\omega + 1)$ with the usual product topology, so that Y is a locally compact, countable metrizable space. Define $f : Y \to Y$ as follows: $f((n,\omega)) = (n+1,\omega)$ for all n in $\mathbb{Z}$, $f((n,m))$ is fixed if $0 < m < |n|$ or $n = m = 0$, $f((n,m)) = f((-n,m))$ for $n = m \in \omega$, and $f((n,m)) = f((n+1,m))$ if $|n| < m$. Then f is an autohomeomorphism of Y, $|\operatorname{orb}((n,\omega))| = \omega$ for every n in $\mathbb{Z}$, but $|\operatorname{orb}((n,m))| < \omega$ for every n in $\mathbb{Z}$ and $m \in \omega$. There is a similar autohomeomorphism on $\mathbb{Q}$.

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