Young Mathematicians Colloquium

20th April 2016

UNIVERSITY OF BIRMINGHAM

PLENARY SPEAKERS

Pure:

Ben Fairbairn
Birbeck University of London

Kevin Hughes
University of Bristol

Katherine Staden
University of Warwick

Applied:

Marie MacCaig
CMAP, École Polytechnique

Paul Roberts
University of Birmingham

Tom Shearer
University of Manchester

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ORGANISERS

Gemma Cupples, Ryan Davies, Raúl Moragues, Chloe Spalding & Alex Tisbury.

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MAGIC

Schedule

Time	Applied (LTC)	Pure (LTA)
9:30 - 10:00	Registration	Registration
10:00 - 11:00	Paul Roberts: Mathematical models of retinal degeneration	Katherine Staden: The Erdös-Rothschild problem on edge-colourings
11:00 - 12:00	Tom Shearer: Microstructual modules of ligament and tendon elasticity and viscoelasticity	Ben Fairbairn: Some thoughts on living and working inside the M25
12:00 - 1:30	Lunch	Lunch
1:30 - 2:30	Marie MacCaig: Tropical algebra: Optimisation, tropical polytopes and integer points	Kevin Hughes: Discrete spherical maximal functions: A duet of the Calderon transference principle and the Hasse principle
2:30 - 3:00	Coffee Break	Coffee Break
3:00 - 3:30	Harvind Rai: Steady-state multiplicity of a Solid Oxide Fuel Cell under a constant external load	Joseph Cook: The Klein Quartic (LTA) Chimere Anabanti: On minimal sizes of locally maximal sum-free sets (LTB)
3:30 - 4:00	James Tyrrell: On the growth of pollen tubes and Finite Volume Methods	Mehsin Atteya: On (σ, τ)-derivation of semiprime rings
4:00 - 4:30	Dominic Henry: Dancing jets and effect of surfactant on inverted thin film flow	Christopher Smithers: Topological machine learning: kernels on point clouds
4:30 - 5:00	Craig Holloway: Linear Taylor-Couette stability of a transversely-isotropic fluid	Tassio Naia: Universal trees in Tournaments

Plenary talks

Some thoughts on living and working inside the M25

Ben Fairbairn
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In 1943/44 Patrick Abercrombie formulated plans for the post-war reconstruction of Londons road network. At the time these were largely ignored but by the 1960s, the unexpected explosion in car ownership lead to the ambitious Ringways Project. This had many consequences for other aspects of town planning at the time including various planned extensions to the M11 to either Brentwood, the never-built Maple Sands airport or Chelmsford as well as a short stretch of motorway just south of Southend. In this talk we will discuss various aspects of these and related constructions, most notably potential extensions to them.

Discrete spherical maximal functions: a duet by the Calderon transference principle and the Hasse principle

Kevin Hughes
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In this talk, we will discuss the discrete spherical averages of Magyar–Stein–Wainger and their ergodic incarnations. Versions of the MSW discrete spherical maximal function over restricted sets of radii bring to light interesting arithmetic phenomena, including a connection to Hasse's principle. This is joint work with Jim Wright and Jacek Zienkiewicz.

Tropical algebra: optimisation, tropical polytopes and integer points

Marie MacCaig
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The tropical semiring is $(R, \oplus, \otimes)$ where $R = \mathbb{R} \cup \{-\infty\}$, $a \oplus b := max(a, b)$ and $a \otimes b := a + b$. These operations are extended to vectors and matrices as in conventional algebra. The tropical linear programming problem seeks to minimise or maximise $cT \otimes x$ subject to constraints $A \otimes x = b$ or $A \otimes x = B \otimes x$. We consider the existence and description of integer solutions to this problem. Additionally, we discuss the problem of determining whether a tropical polytope (a tropical convex hull) contains an integer point. For both of these problems pseudopolynomial algorithms exist but the complexity is unknown. Finally we show that determining the volume of a tropical polytope is NP-hard.

Mathematical models of retinal degeneration

Paul Roberts
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The retina is the layer of tissue at the back of the eye that is responsible for the detection of light. It contains numerous cell types, most notably the photoreceptors, which capture light, converting visual information into an electrical signal, which is transmitted to and interpreted by the brain. Photoreceptors can be characterised as either rods or cones. Rods provide achromatic vision under low light conditions, whilst cones provide high-acuity colour vision under well-lit conditions.

The term Retinitis Pigmentosa (RP) refers to a range of genetically mediated retinal diseases that cause the loss of photoreceptors and hence visual function. RP leads to a patchy degeneration of photoreceptors and typically directly affects either rods or cones, but not both. During the course of the disease, degenerate patches spread and the photoreceptor type unaffected by the mutation also begins to degenerate. The cause underlying these phenomena is currently unknown.

The oxygen toxicity hypothesis suggests that secondary photoreceptor loss is due to hyperoxia (toxically high oxygen levels), which results from the decrease in oxygen uptake following the initial loss of photoreceptors. We have constructed mathematical models, formulated as systems of partial differential equations in 1D and 2D, to investigate this hypothesis.

Using a combination of numerical simulations and mathematical analysis, we find that degeneration may spread due to hyperoxia, replicating some of the spatio-temporal patterns of degeneration seen in vivo. We determine the conditions under which a degenerate patch will spread or remain stable and show that the wave speed of degeneration is a decreasing

function of the local photoreceptor density. Lastly, the effects of treatment with antioxidants and trophic factors, and of capillary loss, upon the dynamics of photoreceptor loss and recovery are also considered.

Microstuctural models of ligament and tendon elasticity and viscoelasticity

Tom Shearer
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Ligaments and tendons are made of collagen fibres organised in a hierarchical structure. Their main subunit is the fascicle, which consists of a large number of fibrils that exhibit varying levels of crimp. In this talk, I will discuss two microstructural models that describe the mechanical behaviour of ligaments and tendons. The first model is a non-linear elastic model, which is expected to be valid in the low strain-rate limit, where hysteresis is minimised. I will derive a new strain energy function for modelling ligaments and tendons based on the geometrical arrangement of their fibrils (which are individually assumed to be linear elastic), and will compare the ability of the new model to reproduce experimental data with that of the commonly-used Holzapfel-Gasser-Ogden (HGO) model. The second model is a viscoelastic model. By assuming that each fibril is linearly viscoelastic, I will show that several complex, non-linear viscoelastic effects can be explained solely by the distribution of the fibril crimp lengths. The viscoelastic model also shows excellent agreement with experimental data, and can reproduce different data sets with the same set of constitutive parameters simply by changing the distribution of the crimp lengths.

The Erdős-Rotschild problem on edge-colourings

Katherine Staden
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Let $\boldsymbol{k} = (k_1, \ldots, k_s)$ be an s-tuple of positive integers. Given a graph G, how many ways are there to colour the edges of G with s colours so that there is no c-coloured copy of the complete graph on k_c vertices, for any $c \in [s]$? Write $F(G; \boldsymbol{k})$ for this quantity and let $F(n; \boldsymbol{k})$ be its maximum over all graphs G on n vertices. What is $F(n, \boldsymbol{k})$ and which graphs G attain this maximum? This problem was first considered by Erdős and Rothschild in 1974, but it has only been solved for a very small number of non-trivial cases. In this talk I will survey the history of the problem, and will discuss some recent general results with Oleg Pikhurko (Warwick) and Zelealem Yilma (Addis Ababa).

Short talks

Applied Speakers

Dancing jets and effect of surfactant on inverted thin film flow

Dominic Henry
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We investigate the stability of a thin film containing insoluble surfactant that is flowing along the underside of an inclined plane, sloped at an angle , taking the small limit so that the incline is near horizontal. Two cases are examined; as well as a single layered film, a two-layer film is considered whereby each liquid layer contains surfactant. The waveless solution in both cases is perturbed and a linear stability analysis conducted, with a discussion on the corresponding growth rates of the perturbations. This work is then compared to an experimental investigation, whereby the underside of a slide fed coating die is considered as the inclined plane, and a series of liquid threads are formed with a uniform spacing. This thread spacing is compared to the most unstable mode of the preceding stability analysis, and compared to the classical Rayleigh-Taylor wavelength which considers a constant surface tension. Moreover, experimentally under certain conditions, the liquid jets were found to oscillate in the transverse direction, or dance. We shall also investigate examples of this phenomenon.

Linear Taylor-Couette stability of a transversely-isotropic fluid

Craig Holloway
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Fibre-laden fluids are found in a variety of situations, whilst Couette devices are used for flow spectroscopy of long biological molecules, such as DNA and proteins in suspension. The presence of these fibres can significantly alter the rheology of the fluid, and hence must be incorporated in any modelling undertaken.

A transversely-isotropic fluid treats these suspensions as a continuum with an evolving preferred direction, through a modified stress tensor incorporating four viscosity-like parameters. We consider the axisymmetric linear stability of a transversely-isotropic viscous fluid, contained between two rotating co-axial cylinders, and determine the critical wave and Taylor numbers for varying gap width and inner cylinder velocity (assuming the outer cylinder is fixed).

Through the inclusion of transversely isotropic effects, the onset of instability is delayed, increasing the range of stable operating regimes. This effect is felt most strongly through incorporation of the anisotropic shear viscosity, although the anisotropic extensional viscosity also contributes. The changes to the rheology induced by the presence of the fibres therefore significantly alter the dynamics of the system, and hence should not be neglected.

Steady-state multiplicity of a Solid Oxide Fuel Cell under a constant external load

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Solid Oxide Fuel Cells (SOFCs) are high temperature fuel cells which converts the chemical energy of Hydrogen (via electrochemical reactions involving Oxygen) into electricity. Multiple steady-states in a Solid Oxide Fuel Cell can arise due to the complex, non-linear relationship between the conductivity of the electrolyte with cell temperature and the reaction rates, leading to the formation of a 'hot spot' within the cell.

The model considered is a planar, zero-dimensional model. The SOFC model takes account of the electrochemical processes within the cell (production/consumption rates, cell voltage, polarizations within the cell), temperature of the solid cell component, diffusion within the electrodes, and gas flow dynamics including mass, momentum and heat transfer. The steady-state model is solved in MATLAB for a wide range of values of the load resistance.

Up to three steady-state solutions can be obtained for a certain region of the external load resistance (two stable, one unstable). The multiplicity region in a fuel cell is shown by the existence of a hysteresis loop in the cell temperature profile. The existence of more than one steady-state solution is backed up by looking into the number of intersections of the heat

production and heat removal curves for values of the load resistance where more than one steady-state solution exists.

On the growth of pollen tubes and finite volume methods

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Modelling the growth of pollen tubes in plants presents a fascinating problem for mathematicians. In order to sustain the rapid expansion of the growing apical area of the tube, sufficient quantities of cell membrane and cell wall material must be transported to this region. This occurs via a combination of long-distance shuttling and short-distance targeting of vesicles containing the requisite materials. Much of the information regarding the specifics of short-distance targeting and the process of exo/endocytosis remains unclear. In this talk, we discuss these processes in more detail and consider how to begin constructing a model for growth, using Dumais model linking elements of mechanics, chemistry and hydraulics as an example. We then spend some time looking at our development of finite volume methods in Matlab, which we hope to later use to create an accurate spatiotemporal model of pollen tube growth.

Pure Speakers

On minimal sizes of locally maximal sum-free sets

Chimere Anabanti
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In 1985, Babai and Sos asked the question: can we bound the minimal size of a locally maximal sum-free set in a finite group G? In this talk, we answer in affirmation for elementary abelian 2-groups. Our results here are also relevant to finite geometers who study the packing problem: determination of minimal sizes of complete caps in the projective space PG(n-1, 2).

On (σ, τ)-derivation of semiprime rings

Mehsin Jabel Atteya
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Joint with Dalal Ibraheem Rasen. In this paper we investigate some results about the commute additive mappings a (σ, τ)-derivation on semiprime ring R. Throughout R will represent an assosiative ring with center Z. Recall that a ring R is prime if $xRy = 0$ implies $x = 0$ or $y = 0$, and semiprime if $xRx = 0$ implies $x = 0$. In fact, a prime ring is semiprime but the converse is not true in general. An additive mapping $d : R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. Let σ and τ be two automorphisms of R. An additive mapping $d : R \to R$ is said to be a (σ, τ)-derivation if $d(xy) = d(x)\sigma(y) + \tau(x)d(y)$ holds for all $x, y \in R$. Following we mention some results of the paper. Lemma: Let R be a semiprime ring and σ and τ be two automorphisms mappings of R. Suppose there exist a (σ, τ)-derivation d such that $d^2(R) = 0$ for all $x \in R$ and d is commute with both σ, τ then $d = 0$. Theorem: Let R be a 2-torsion free semiprime ring and σ, τ be automorphisms mappings of R. Suppose there exist a (σ, τ)-derivation d such that $[d(x), x]\sigma, \tau = 0$ for all $x \in R$. Then d is commuting mapping of R.

The Klein Quartic

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The Klein quartic is the most symmetric compact Riemann surface of genus 3. I will explain these concepts in the talk. I will discuss the concept of a pants decomposition, and show the Fenchel-Nielsen coordinates of the Klein quartic, which I have recently calculated. I will also show some eigenvalues and eigenfunctions that I have computed. If time permits, I can discuss some other interesting properties of the surface.

Universal trees in Tournaments

Tássio Naia
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An oriented tree T on n vertices is *unavoidable* if every n-vertex tournament contains a copy of T. Alon raised the question of which trees are unavoidable. Only a few examples of such trees are known, such as oriented paths (as demonstrated by Thomason). We exhibit a large class of unavoidable trees, including almost all orientations of large balanced q-ary trees for any fixed q.

This is joint work with Richard Mycroft.

Topological machine learning: Kernels on point clouds

Christopher Smithers
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Over the past few years, techniques from algebraic topology have been adapted so as to be usable in helping to understand the "shape" of data, giving rise to the field of topological data analysis. Persistent homology is a tool for finding topological features of a point cloud which persist at multiple scales, giving an indication of the topological properties of the underlying space the points are assumed to approximate. Whilst these persistent features are well understood (as "persistence diagrams" or "barcodes") , there is no comparison between clouds. In fact, by comparing clouds, and looking at differences and similarities in their persistence barcodes, we can hope to learn more about the individual clouds, and classify them more robustly.

In particular, I will discuss current work in using these barcodes as the basis for a number of kernels. Kernels are essentially a measure of similarity between objects, and are used primarily as the input of a machine learning technique known as Support Vector Machine Learning. I will survey some previous progress in developing such kernels, and compare them with a number of proposed alternatives. Our ultimate aim is to be able to automatically detect the topological structures of a large collection of point clouds, grouping those clouds into different classes based on their 'shape'. For example, can we distinguish between clouds sampling a sphere, and those sampling a torus? What about points on a circle vs points on a square?

List of Participants

- Mohammed Alaa Abdulameer, University of Leicester
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- Chimere Anabanti, Birkbeck University of London
- Mehsin Jabel Atteya, University of Leicester
- David Beltran, University of Birmingham
- Joseph Cook, Loughborough University
- Gemma Cupples, University of Birmingham
- Ryan Davies, University of Birmingham
- Gaurav Dhariwal, University of York
- Ben Fairbairn, Birkbeck University of London
- Meurig Gallagher, University of Birmingham
- Stefan Glock, University of Birmingham
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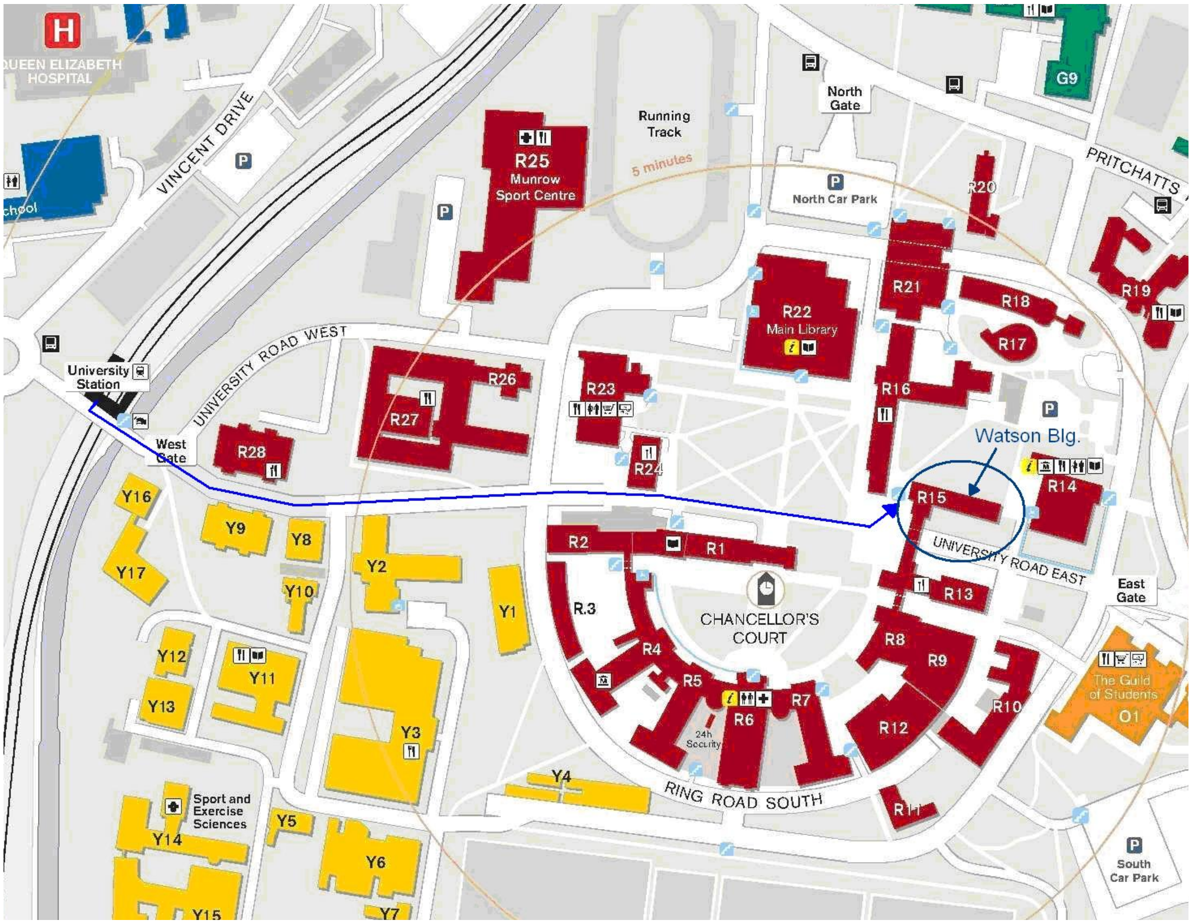
- James Tyrrell, University of Birmingham
- Athar Warraich, University of Birmingham
- James Whitley, University of Birmingham
- Alec Wilson, University of Bristol

University of Birmingham Map of Campus

R15: Watson Building (School of Mathematics)
R24: Staff House (Lunch location)



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To claim reimbursement for travel costs/accommodation, please fill in the contained form and send, along with any train tickets/receipts, to:

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Notes